Fall 2013 **Exam #3** COSC 362

Name: _____

Write all of your responses on the exam paper or on the extra paper provided. Turn in all work and this exam paper.

1. (*20 Points*) Construct an NPDA that accepts the language $L = \{w \mid n_a(w) < n_b(w)\}$.

2. (*20 Points*) Convert the following CFG to an NPDA.

$$\begin{aligned} S &\longrightarrow aABb \\ A &\longrightarrow bBA \,|\, aA \,|\, aAA \,|\, b \\ B &\longrightarrow aABa \,|\, a \end{aligned}$$

3. (*25 Points*) Show that the language $L = \{a^n b^t c^n \mid t > n\}$ is not context free.

4. **True & False:** (*20 Points*) Mark each of the following as being either true or false.

(a) _____ Any language that can be represented as the concatenation of a context-free language and a regular language can be accepted by a DPDA.

(b) _____ The intersection of two context-free languages is context-free.

(c) _____ The complement of a deterministic context-free language is deterministic context-free.

(d) _____ The star closure of a context-free language is context-free.

(e) _____ The union of a context-free language with a regular language is regular.

(f) _____ The complement of a regular language is deterministic context-free.

(g) _____ The concatenation of a context-free language and a regular language is context-free.

(h) _____ The complement of a context-free language can be represented as a finite union of context-free languages.

(i) _____ In order for a language to be non-context-free the alphabet of that language must contain at least 3 distinct characters.

(j) _____ The intersection of a context-free language and a regular language is context-free.

(k) _____ The union of two deterministic context-free languages is deterministic context-free.

(l) _____ The intersection of two deterministic context-free languages is deterministic context-free.

(m) _____ If L_1 is context free and L_2 is regular then $L_1 - L_2$ is context-free.

(n) _____ If L_1 is deterministic context-free and L_2 is regular then $L_1 - L_2$ is deterministic context-free.

(o) _____ The union of two unambiguous context-free languages is an unambiguous context-free language.

(p) _____ The intersection of two unambiguous context-free languages is an unambiguous context-free language.

(q) _____ The language

$$L = \{w \mid n_a(w) = n_b(w) \text{ and } w \text{ does not contain the substring } aab\}$$

is context-free.

(r) _____ The language $L = \{a^n b^k c^t \mid t = k \text{ or } t = 2k\}$ is context-free.

(s) _____ The language $L = \{wcw^R \mid w \in \{a, b\}^*\}$ is deterministic context-free.

(t) _____ The language $L = \{a^n b^k \mid n \leq k^2\}$ is context-free.

5. (*25 Points*) Construct a standard Turing Machine by displaying the set of transitions for the Turing Machine that will copy a word $w \in \{a, b\}^*$ in reverse. Specifically, given w on the tape with the read/write head on the last letter of the word, the machine will produce either $w\Box w^R$ or ww^R on the tape, your choice. It is assumed that there are only blanks after w on the tape when the machine starts.