Fall 2013

Exam #3 Key

COSC 362

1. (*20 Points*) Construct an NPDA that accepts the language $L = \{w \mid n_a(w) < n_b(w)\}$.

   Solution:

$$\begin{aligned}
\delta(q_0, a, z) &= \{(q_0, 0z)\} \\
\delta(q_0, b, z) &= \{(q_0, 1z)\} \\
\delta(q_0, a, 0) &= \{(q_0, 00)\} \\
\delta(q_0, b, 1) &= \{(q_0, 11)\} \\
\delta(q_0, a, 1) &= \{(q_0, \lambda)\} \\
\delta(q_0, b, 0) &= \{(q_0, \lambda)\} \\
\delta(q_0, \lambda, 1) &= \{(q_f, \lambda)\}
\end{aligned}$$

2. (*20 Points*) Convert the following CFG to an NPDA.

$$\begin{aligned}
S &\longrightarrow aABb \\
A &\longrightarrow bBA \mid aA \mid aAA \mid b \\
B &\longrightarrow aABa \mid a
\end{aligned}$$

   Solution: First do a minor conversion to Greibach normal form,

$$\begin{aligned}
S &\longrightarrow aABD \\
A &\longrightarrow bBA \mid aA \mid aAA \mid b \\
B &\longrightarrow aABC \mid a \\
C &\longrightarrow a \\
D &\longrightarrow b
\end{aligned}$$

   Now apply the conversion algorithm to convert to an NPDA,

$$\begin{aligned}
\delta(q_0, \lambda, z) &= \{(q_1, Sz)\} \\
\delta(q_1, a, S) &= \{(q_1, ABD)\} \\
\delta(q_1, b, A) &= \{(q_1, BA)\} \\
\delta(q_1, a, A) &= \{(q_1, A)\} \\
\delta(q_1, a, A) &= \{(q_1, AA)\} \\
\delta(q_1, b, A) &= \{(q_1, \lambda)\} \\
\delta(q_1, a, B) &= \{(q_1, ABC)\} \\
\delta(q_1, a, B) &= \{(q_1, \lambda)\} \\
\delta(q_1, a, C) &= \{(q_1, \lambda)\} \\
\delta(q_1, b, D) &= \{(q_1, \lambda)\} \\
\delta(q_1, \lambda, z) &= \{(q_f, z)\}
\end{aligned}$$

3. (*25 Points*) Show that the language $L = \{a^n b^t c^n \mid t > n\}$ is not context free.

Solution: By way of contradiction assume that L is context-free. Then by the context-free pumping lemma we know that there exists a fixed positive integer m such that any word $w \in L$, with length $|w| \geq m$, can be written as the concatenation $w = uvxyz$ with vy not empty, $|vxy| \leq m$ and $uv^i xy^i z \in L$ for each $i \geq 0$. Let us choose the word

$$w = a^m b^{m+1} c^m$$

where m is the positive integer guaranteed by the pumping lemma for language L. Since vy is not empty and $|vxy| \leq m$ we know that either $vy = a^j$, $vy = a^j b^r$, $vy = b^j$, $vy = b^j c^r$ or $vy = c^j$ where $0 < j, r \leq m$. We have the following cases,

(a) If $vy = a^j$ then by the pumping lemma $uv^2xy^2z = a^{m+j}b^{m+1}c^m \in L$ which is absurd since $m + j \neq m$.

(b) If $vy = a^j b^r$ then by the pumping lemma $uv^2xy^2z = a^{m+j}b^{m+1+r}c^m \in L$ which is absurd since $m + j \neq m$.

(c) If $vy = b^j$ then by the pumping lemma $uv^0xy^0z = a^m b^{m+1-j} c^m \in L$ which is absurd since $m + 1 - j \leq m$.

(d) If $vy = b^j c^r$ then by the pumping lemma $uv^2xy^2z = a^m b^{m+1+j} c^{m+r} \in L$ which is absurd since $m + r \neq m$.

(e) If $vy = c^j$ then by the pumping lemma $uv^2xy^2z = a^m b^{m+1} c^{m+j} \in L$ which is absurd since $m + j \neq m$.

Hence, the language L is not context-free.

4. **True & False:** (*20 Points*) Mark each of the following as being either true or false.

   (a) **FALSE:** Any language that can be represented as the concatenation of a context-free language and a regular language can be accepted by a DPDA.

   (b) **FALSE:** The intersection of two context-free languages is context-free.

   (c) **FALSE:** The complement of a deterministic context-free language is deterministic context-free.

   (d) **TRUE:** The star closure of a context-free language is context-free.

   (e) **FALSE:** The union of a context-free language with a regular language is regular.

   (f) **TRUE:** The complement of a regular language is deterministic context-free.

   (g) **TRUE:** The concatenation of a context-free language and a regular language is context-free.

   (h) **FALSE:** The complement of a context-free language can be represented as a finite union of context-free languages.

   (i) **FALSE:** In order for a language to be non-context-free the alphabet of that language must contain at least 3 distinct characters.

   (j) **TRUE:** The intersection of a context-free language and a regular language is context-free.

   (k) **FALSE:** The union of two deterministic context-free languages is deterministic context-free.

   (l) **FALSE:** The intersection of two deterministic context-free languages is deterministic context-free.

   (m) **TRUE:** If L_1 is context free and L_2 is regular then $L_1 - L_2$ is context-free.

   (n) **TRUE:** If L_1 is deterministic context-free and L_2 is regular then $L_1 - L_2$ is deterministic context-free.

   (o) **FALSE:** The union of two unambiguous context-free languages is an unambiguous context-free language.

   (p) **FALSE:** The intersection of two unambiguous context-free languages is an unambiguous context-free language.

   (q) **TRUE:** The language

   $$L = \{w \mid n_a(w) = n_b(w) \text{ and } w \text{ does not contain the substring } aab\}$$

   is context-free.

   (r) **TRUE:** The language $L = \{a^n b^k c^t \mid t = k \text{ or } t = 2k\}$ is context-free.

   (s) **TRUE:** The language $L = \{wcw^R \mid w \in \{a, b\}^*\}$ is deterministic context-free.

   (t) **FALSE:** The language $L = \{a^n b^k \mid n \leq k^2\}$ is context-free.

5. (*25 Points*) Construct a standard Turing Machine by displaying the set of transitions for the Turing Machine that will copy a word $w \in \{a, b\}^*$ in reverse. Specifically, given w on the tape with the read/write head on the last letter of the word, the machine will produce either $w\Box w^R$ or ww^R on the tape, your choice. It is assumed that there are only blanks after w on the tape when the machine starts.

Solution: This machine will convert w to ww^R, the final position of the read/write head is on the last character of ww^R.

$$
\begin{aligned}
\delta(q_0, a) &= (q_a, x, R) \\
\delta(q_0, b) &= (q_b, y, R) \\
\delta(q_0, \Box) &= (q_f, \Box, R) \\
\delta(q_a, x) &= (q_a, x, R) \\
\delta(q_a, y) &= (q_a, y, R) \\
\delta(q_a, \Box) &= (q_r, x, L) \\
\delta(q_b, x) &= (q_b, x, R) \\
\delta(q_b, y) &= (q_b, y, R) \\
\delta(q_b, \Box) &= (q_r, y, L) \\
\delta(q_r, x) &= (q_r, x, L) \\
\delta(q_r, y) &= (q_r, y, L) \\
\delta(q_r, a) &= (q_a, x, R) \\
\delta(q_r, b) &= (q_b, y, R) \\
\delta(q_r, \Box) &= (q_c, \Box, R) \\
\delta(q_c, x) &= (q_c, a, R) \\
\delta(q_c, y) &= (q_c, b, R) \\
\delta(q_c, \Box) &= (q_f, \Box, L)
\end{aligned}
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