Fall 2013 — COSC 362

Final Exam

Name: ______________________________

Write all of your responses on the exam paper or on the extra paper provided. Turn in all work and this exam paper.

1. **Definitions & Short Answer:** (*25 Points*)

   (a) Discuss the primary differences between Finite Automata, Pushdown Automata and Turing Machines as far as models of computation are concerned.

   (b) What is a Leftmost derivation?

(c) Define a functional property.

(d) What does it mean for a language to be inherently ambiguous?

(e) State the Turing's Thesis.

2. **True & False:** (*40 Points*) Mark each of the following as being either true or false.

(a) ________ Any language that can be represented as the concatenation of a context-free language and a regular language can be accepted by an NPDA.

(b) ________ Regular languages are closed under union, intersection, star closure, complementation, difference and concatenation.

(c) ________ The complement of a deterministic context-free language is context-free but it might not be deterministic.

(d) ________ The context-free languages are closed under union, concatenation and star closure.

(e) ________ The complement of a regular language is deterministic context-free.

(f) ________ The complement of a context-free language can be represented as a union of context-free languages.

(g) ________ The intersection of $L_1 = \{a^n b^k a^t \mid t = 2k \text{ or } t = 3n\}$ and $L_2 = \{a^n b^k a^t \mid n, k, t \geq 3\}$ is context-free.

(h) ________ The union of two deterministic context-free languages is deterministic context-free.

(i) ________ The intersection of two deterministic context-free languages is deterministic context-free.

(j) ________ If deterministic context-free languages are closed under regular difference.

(k) ________ The language

$$L = \{w \mid n_a(w) > 5n_b(w) \text{ and } w \text{ does not contain any substring of the form } ba^*b\}$$

is context-free.

(l) ________ The class of decidable languages is closed under complementation.

(m) ________ The class of semidecidable languages is closed under complementation.

(n) ________ If both L and $\overline{L}$ are semidecidable then L is decidable.

(o) ________ If not all languages are semidecidable.

(p) ________ Semidecidable languages are closed under union and intersection.

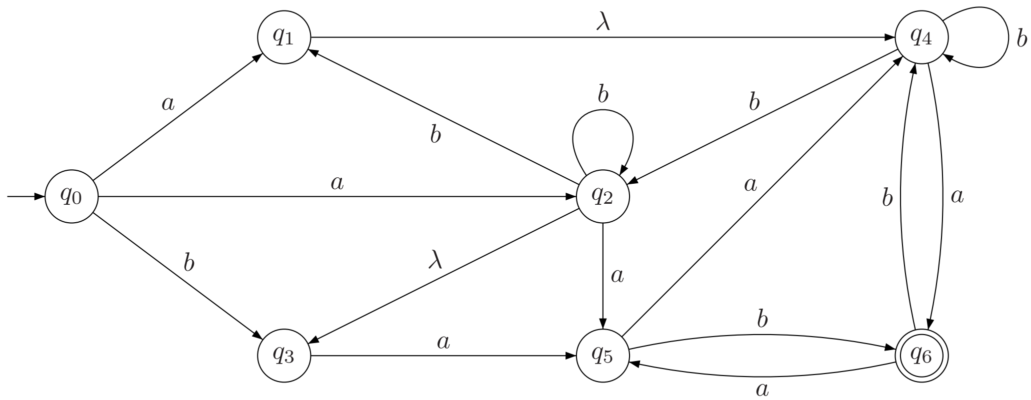
(q) ________ All context-free languages are semidecidable.

(r) ________ Semidecidable languages are closed under regular difference.

(s) ________ The language $L = \{P \mid P \text{ halts in 10 or fewer steps on every input}\}$ defines a functional property.

(t) ________ The language $L = \{P \mid P \text{ is equivalent to a given program } Q\}$ defines a functional property.

3. **Finite Automata:** (*30 Points*) Consider the following NFA, A.



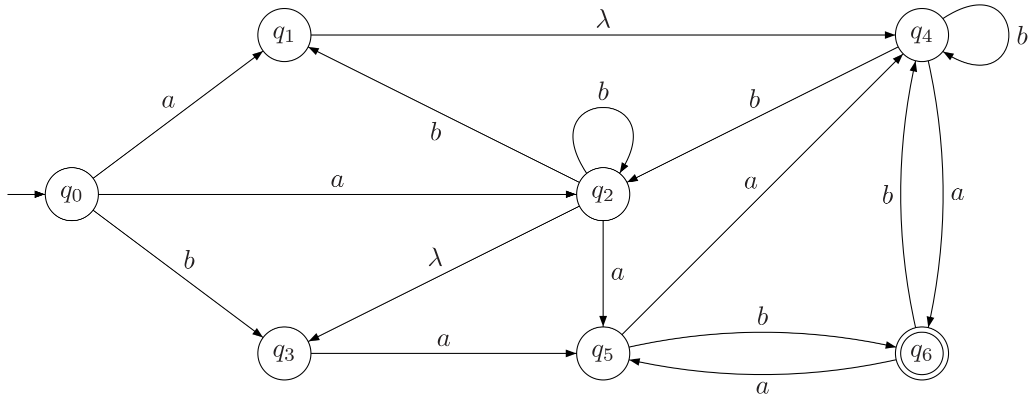
(a) Determine if the automaton accepts the following words. If it does, display the sequence of states that drive the word to a final state.

i. Is *abbbb* acceptable or not acceptable? If it is acceptable display a sequence of states for the word that end in a favorable state. If it is not acceptable give a *short* explanation why.

ii. Is *abbbabaaa* acceptable or not acceptable? If it is acceptable display a sequence of states for the word that end in a favorable state. If it is not acceptable give a *short* explanation why.

iii. Is $aaa^* \subset L(A)$? Prove or disprove your answer.

(b) Convert this NFA to a DFA.



4. **Context-Free Languages, Grammars & Push-Down Automata:** (*30 Points*) This exercise deals with the language $L = \{a^k b^n c\, b^n a^k \mid n, k > 0\}$.

   (a) Construct a context-free grammar for the language L.

   (b) Convert your grammar from 4a to Chomsky Normal Form.

(c) Construct a deterministic push-down automaton that accepts the language $L = \{a^k b^n c\, b^n a^k \mid n, k > 0\}$.

5. **Turning Machines:** (*20 Points Each*) Do each of the following.

   (a) Construct a standard Turing Machine by displaying the set of transitions for the Turing Machine that will take as input a unary number and return its binary equivalent. That is, it converts unary to binary. For example, an input of $\underline{1}11111$ will produce an output of $\underline{1}10$ and an input of $\underline{1}1111111111$ will produce an output of $\underline{1}011$.

(b) Use the primitives R, L, R_a, L_a, R_b, L_b, $R_\Box$, $L_\Box$, R_0, L_0, R_1, L_1, $R_{\overline{a}}$, $L_{\overline{a}}$, $R_{\overline{b}}$, $L_{\overline{b}}$, $R_{\overline{\Box}}$, $L_{\overline{\Box}}$, $R_{\overline{0}}$, $L_{\overline{0}}$, $R_{\overline{1}}$, $L_{\overline{1}}$, a, b, 0, 1, $\Box$, A, S, Shl, Shr, N_L, N_R, W_E and W_B, and the tape alphabet of $\{a, b, 0, 1, \Box\}$ where,

A — Adds one in binary, the read/write head begins and ends on the leftmost digit. So applying it to $\underline{1}00101$ produces $\underline{1}00110$. Also the number grows to the left, so $\Box\underline{1}11$ produces $\underline{1}000$.

S — Subtracts one in binary, the read/write head begins and ends on the leftmost digit. So applying it to $\underline{1}00110$ produces $\underline{1}00101$. Also the number shrinks on the left, so $\underline{1}000$ produces $\Box\underline{1}11$.

Shl — Shifts a word one space to the left. So $\Box\underline{a}ba$ produces $\underline{a}ba\Box$.

Shr — Shifts a word one space to the right. So $\underline{a}ba\Box$ produces $\Box\underline{a}ba$.

N_L — Moves the read/write head to the beginning of the next word to the left.

N_R — Moves the read/write head to the beginning of the next word to the right.

W_E — Moves the read/write head to the end of the word. If the read/write head is on a space the head does not move.

W_B — Moves the read/write head to the beginning of the word. If the read/write head is on a space the head does not move.

Construct a Turing machine (in diagram form) that will take an input of a single word from $\{a, b\}^*$ and write the number of a's in binary on front of the word. The original word is not altered by the computation. For example, if the input tape is $\Box\underline{a}bbbababbaa\Box$ the Turing machine produces $\Box101\Box\underline{a}bbbababbaa\Box$.

6. **Proofs of Membership and Non-membership:** (*20 Points*) Do one and only one of the following.

   (a) Prove that the language $L = \{a^k b^{3k} c^{2k} \mid k = 0, 1, 2, 3, \ldots\}$ is not context-free.

   (b) Prove that the language $L = \{P \mid P$ does not halt on an input of $2\}$ is undecidable.

   (c) Prove that the language $L = \{a^n b^k \mid \frac{n}{k}$ is an integer$\}$ is not regular.

7. **Proofs of General Results:** (*25 Points*) Do one and only one of the following.

(a) Prove that the cardinality of the set of all Turing machines is countable.

(b) Prove that there exists a function $f : \mathbb{N} \to \mathbb{N}$ that is not partial Turing computable.

(c) Define the Halt (H) program as we did in class and then prove that it does not exist.