Name: _____

Write all of your responses on the extra paper provided. Hand in this exam paper along with your solutions, please place your name on the top of each page. Show all of your work.

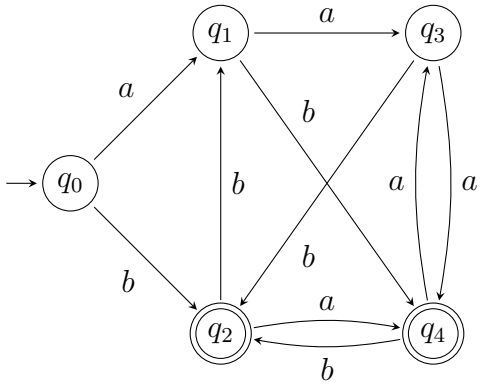
1. (20 Points) Answer each of the following questions on languages and grammars. For this exercise, $\Sigma = \{a, b\}$.

   (a) Find a grammar that generates the language of all strings that have length of at least 4 and the first two characters are opposites of the last two characters. So if the word begins with *aa* it must end with *bb*, if it begins with *ba* it must end with *ab*, and so on. For example, the words *aaabbaabb*, *abbaaba*, *baabab*, and *bbabbaa* are all in the language.

   (b) Find a grammar that generates the language,

   $$L = \{a^n b^m a^{2m} \mid n \geq 0 \text{ and } m \geq 0\}$$

2. (20 Points) Consider the following DFA, M



   (a) Determine if the automaton accepts the following words. Display the sequence of states for each word.

      i. *abbaba*

      ii. *bbbbb*

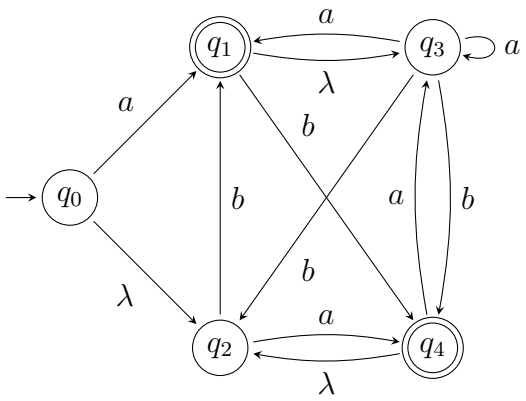
   (b) If $a^n \in L(M)$ then what are all of the possible values of n.

   (c) If $b^n \in L(M)$ then what are all of the possible values of n.

3. (10 Points) Construct a DFA, with $\Sigma = \{a, b\}$, that accepts the language,

   $$L = \{abwa^n \mid n \geq 2 \text{ and } w \in \{a, b\}^*\}$$

4. (20 Points) Consider the following NFA, M



   (a) Convert the NFA to a DFA

   (b) Describe $L(M)$.

5. (10 Points) Give a regular expression for the language,

$$L = \{abwa^n \mid n \geq 2 \text{ and } w \in \{a, b\}^*\}$$

6. (10 Points) Give a regular expression for the language of all strings that contain at least one occurrence of each symbol in $\Sigma = \{a, b, c\}$

7. (20 Points) Convert the following NFA to a regular expression.