1. (20 Points) Answer each of the following questions on languages and grammars. For this exercise, $\Sigma = \{a, b\}$.

   (a) Find a grammar that generates the language of all strings that have length of at least 4 and the first two characters are opposites of the last two characters. So if the word begins with *aa* it must end with *bb*, if it begins with *ba* it must end with *ab*, and so on. For example, the words *aaabbaabb*, *abbaaba*, *baabab*, and *bbabbaa* are all in the language.

   Solution:

   $$\begin{aligned} S &\rightarrow aaAbb \\ S &\rightarrow abAba \\ S &\rightarrow baAab \\ S &\rightarrow bbAaa \\ A &\rightarrow aA \\ A &\rightarrow bA \\ A &\rightarrow \lambda \end{aligned}$$

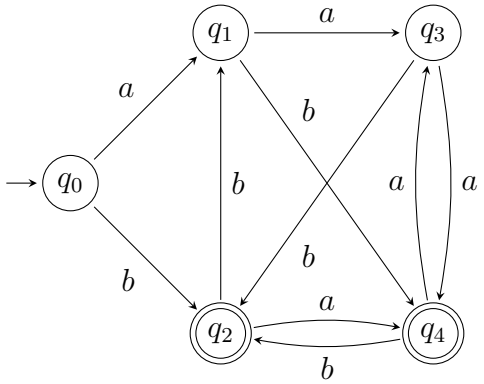
   (b) Find a grammar that generates the language,

   $$L = \{a^n b^m a^{2m} \mid n \geq 0 \text{ and } m \geq 0\}$$

   Solution:

   $$\begin{aligned} S &\rightarrow AB \\ A &\rightarrow aA \\ B &\rightarrow bBaa \\ A &\rightarrow \lambda \\ B &\rightarrow \lambda \end{aligned}$$

2. (20 Points) Consider the following DFA, M



   (a) Determine if the automaton accepts the following words. Display the sequence of states for each word.

   i. *abbaba*
   Solution: *abbaba* — $q_0, q_1, q_4, q_2, q_4, q_2, q_4$, Accepted

   ii. *bbbbb*
   Solution: *bbbbb* — $q_0, q_2, q_1, q_4, q_2, q_1$, Not Accepted

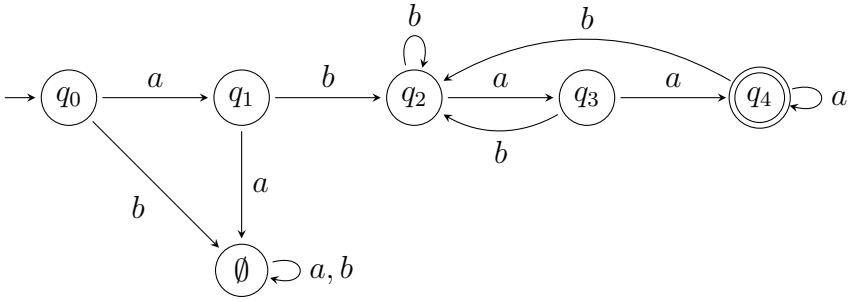
(b) If $a^n \in L(M)$ then what are all of the possible values of n.
Solution: $n = 3 + 2k$ where $k \geq 0$.

(c) If $b^n \in L(M)$ then what are all of the possible values of n.
Solution: $n = 1, 3, 4, 6, 7, 9, 10, 12, 13, \ldots$, that is, all values except for $n = 2+3k$ where $k \geq 0$.

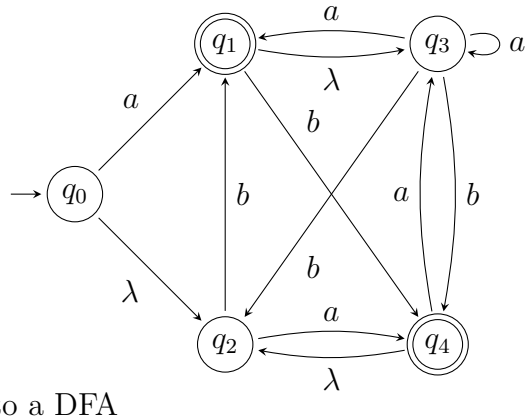
3. (10 Points) Construct a DFA, with $\Sigma = \{a, b\}$, that accepts the language,

$$L = \{abwa^n \mid n \geq 2 \text{ and } w \in \{a, b\}^*\}$$

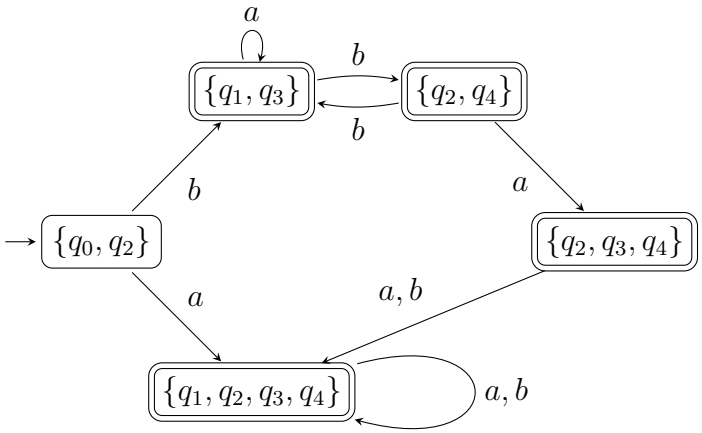
Solution:



4. (20 Points) Consider the following NFA, M



(a) Convert the NFA to a DFA
Solution:



(b) Describe $L(M)$.

Solution:

$L(M) = \Sigma^+$

5. (10 Points) Give a regular expression for the language,

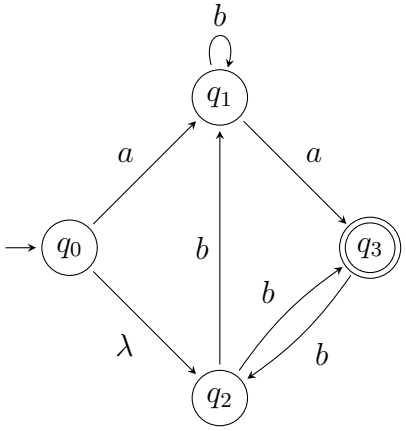
$$L = \{abwa^n \mid n \geq 2 \text{ and } w \in \{a, b\}^*\}$$

Solution: $ab(a+b)^*aaa^*$

6. (10 Points) Give a regular expression for the language of all strings that contain at least one occurrence of each symbol in $\Sigma = \{a, b, c\}$

Solution: $(a+b+c)^*a(a+b+c)^*b(a+b+c)^*c(a+b+c)^* + (a+b+c)^*a(a+b+c)^*c(a+b+c)^*b(a+b+c)^* + (a+b+c)^*b(a+b+c)^*a(a+b+c)^*c(a+b+c)^* + (a+b+c)^*b(a+b+c)^*c(a+b+c)^*a(a+b+c)^* + (a+b+c)^*c(a+b+c)^*a(a+b+c)^*b(a+b+c)^* + (a+b+c)^*c(a+b+c)^*b(a+b+c)^*a(a+b+c)^*$

7. (20 Points) Convert the following NFA to a regular expression.



Solution: $(b + bb^*a + ab^*a)(b(b + bb^*a))^*$