Name: _____

Write all of your responses on the extra paper provided. Hand in this exam paper along with your solutions, please place your name on the top of each page. Show all of your work.

1. (*10 Points*) Give a regular grammar and DFA for the language,

$$L = \{w \mid (n_a(w) - n_b(w)) \mod 3 = 1\}$$

2. (*15 Points Each*) **For any two of the following three languages**, make a conjecture whether or not it is regular. Then prove your conjecture.

   (a) $L = \{a^n b^p \mid |n - p| = 5\}$

   (b) $L = \{vwv \mid v, w \in \{a, b\}^* \text{ and } |v| = 3\}$

   (c) $L = \{a^n \mid n = k^3 \text{ for some integer } k\}$

3. (*15 Points Each*) **For any two of the following three languages**, find context free grammars for them.

   (a) $L = \{a^n b^p c^t \mid p = 2n + 3t\}$

   (b) $L = \{a^n b^p c^t \mid t > n + p\}$

   (c) $L = \{a^n b^p \mid p \leq n \leq 4p\}$

4. (*10 Points*) Show that the following grammar is ambiguous.

$$\begin{aligned} S &\longrightarrow abAB \\ A &\longrightarrow bAB \mid b \mid \lambda \\ B &\longrightarrow BAa \mid A \mid a \mid \lambda \end{aligned}$$

5. (*30 Points*) Consider the following grammar, G. In each conversion step below, follow the conversion or removal algorithm discussed in class.

$$\begin{aligned} S &\longrightarrow abAB \\ A &\longrightarrow BC \mid b \mid \lambda \\ B &\longrightarrow BAa \mid A \mid a \mid \lambda \\ C &\longrightarrow Ab \end{aligned}$$

   (a) Remove all λ-productions.

   (b) Remove all unit-productions from your result in 5a.

   (c) Remove all useless productions from your result in 5b.

   (d) Convert the grammar into Chomsky Normal Form from your result in 5c.