Name: _____

Write all of your responses on the extra paper provided. Hand in this exam paper along with your solutions, please place your name on the top of each page. Show all of your work.

1. **True & False:** (*2 Points Each*) Mark each of the following as being either true or false.

   (a) _____ The union of two deterministic context-free languages is deterministic context-free.

   (b) _____ The star closure of any context free language can be realized as a push-down automaton.

   (c) _____ If L_1 is context free and L_2 is regular then the language $L = L_1 - L_2$ is context-free.

   (d) _____ The intersection of two context-free languages is context-free.

   (e) _____ The complement of a context free language is context free.

2. (*15 Points*) State Turing's Thesis (also known as The Church-Turing Thesis) and explain its meaning.

3. (*15 Points*) Construct an NPDA for the language $L = \{a^n b^m \mid n \leq m \leq 3n, n \geq 0\}$.

4. (*15 Points*) Show that the language $L = \{a^n b^n \mid n \geq 1\} \cup \{a\}$ is deterministic context free.

5. (*15 Points*) Create a context free grammar for the language

   $$L = \{a^n b^m c^{n+m} \mid n \geq 0, m \geq 0\}$$

   and then convert the grammar to an NPDA.

6. (*20 Points*) Show that the language $L = \{w \mid n_a(w) \cdot n_b(w) = n_c(w)\}$ is not context free.

7. (*20 Points*) Create a Turing machine by displaying its set of transitions that will take a binary number on the tape and add one to it. It assumed that there is a number on the tape and that the read/write head is on the far right digit of the number. The machine is to add one to the number on the tape (in binary) and return the read/write head to the far right digit of the number.