Name: ____

You may fill in the True & False on this page but write all of your responses to the other questions on the extra paper provided. Hand in this exam paper along with your solutions, please place your name on the top of each page. Show all of your work.

1. **True & False:** (*2 Points Each*) Mark each of the following as being either true or false.

   (a) ________ The star closure of any context free language can be realized as a push-down automaton.

   (b) ________ The power set of a set can be put into a one-to-one correspondence with a countable union of copies of the original set.

   (c) ________ If L_1 is context free and L_2 is regular then the language $L = L_2 - L_1$ is context-free.

   (d) ________ The union, intersection, concatenation, complement, and star closure of two context-free languages is context-free.

   (e) ________ A nondeterministic push-down automaton is more powerful than a deterministic push-down automaton.

   (f) ________ There are more Turing machines than there are finite automata.

   (g) ________ Decidable languages are closed under complement.

   (h) ________ A nondeterministic Turing machine is more powerful than a deterministic Turing machine.

   (i) ________ The Universal Turing machine can run itself.

   (j) ________ The union of two deterministic context-free languages is deterministic context-free.

   (k) ________ The union of a context-free language with a regular language is context-free.

   (l) ________ The union, intersection, concatenation, complement, and star closure of two regular languages is regular.

   (m) ________ A Turing Machine can recognize any context-free language.

   (n) ________ Gödel numbering is the process of putting the set of all Turing Machines in a one-to-one correspondence with the natural numbers.

   (o) ________ A programming language interpreter can be simulated by a Turing machine.

2. **Definitions & Short Answer:** (*5 Points Each*) Answer all of the following.

   (a) Discuss the primary differences between Finite Automata, Push-Down Automata, and Turing Machines as far as models of computation are concerned.

   (b) State Turing's Thesis (also known as The Church-Turing Thesis) and explain its meaning.

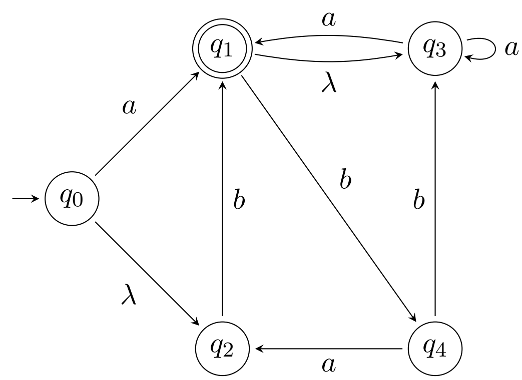
   (c) What does it mean for a language to be ambiguous and inherently ambiguous? Give an example of an inherently ambiguous language.

   (d) Define an algorithm.

   (e) What is the Universal Turing Machine?

   (f) Define Chomsky Normal Form and discuss its significance.

3. **Finite Automata:** (*30 Points*) Consider the following NFA, A.



   (a) What are all the words of length 4 that are accepted by A?

   (b) Convert A to a DFA.

4. **Context-Free Languages, Grammars & Push-Down Automata:** (*30 Points*) Consider the context-free grammar G with $\Sigma = \{a, b\}$ and rules,

$$\begin{aligned} S &\rightarrow abB \mid ab \\ A &\rightarrow bB \mid \lambda \\ B &\rightarrow abB \mid A \mid a \end{aligned}$$

   (a) Remove all λ productions, unit productions, and useless productions.

   (b) Convert the grammar to Chomsky Normal form.

   (c) Convert the grammar to an NPDA.

5. **Turning Machines:** (*15 Points*) Do one of the following.

   (a) Construct a Turing machine, by displaying its set of transitions, that will accept the language $L = \{w \mid n_a(w) = n_b(w)\}$, assume that $\Sigma = \{a, b\}$. Assume that the read/write head is on the far left character of w at the start. The only thing on the tape at the start is the word w with spaces before and after it.

   (b) Construct a Turing machine, by displaying its set of transitions, that will accept the language $L = \{ww^R \mid w \in \{a, b\}^*\}$, assume that $\Sigma = \{a, b\}$. Assume that the read/write head is on the far left character of w at the start. The only thing on the tape at the start is the word w with spaces before and after it.

6. **Turning Machines:** (*15 Points*) Do one of the following.

   (a) Construct a Turing machine, by displaying its set of transitions, that will compute the function, $f(x) = \left\lfloor \frac{x}{2} \right\rfloor$, where x is on the tape in unary form and there is a blank before and after x. Assume that the read/write head is on the far left digit of x at the start and return it there when the machine is finished.

   (b) Create a Turing machine, by displaying its set of transitions, that will take a binary number on the tape and subtract one from it. It assumed that there is a number on the tape that is greater than or equal to 1, and that the read/write head is on the far right digit of the number. The machine is to subtract one from the number on the tape (in binary) and return the read/write head to the far right digit of the number.

7. **Membership and Non-membership:** (*15 Points*) Do one of the following.

   (a) Prove or disprove that the language $L = \{a^n b^{3n} \mid \text{where } n \text{ is a positive integer}\}$ is regular.

   (b) Prove or disprove that the language $L = \{a^{2^n} \mid \text{where } n \text{ is a positive integer}\}$ is regular.

8. **Membership and Non-membership:** (*15 Points*) Do one of the following.

   (a) Prove or disprove that the language $L = \{a^n b^{n^2} c^{n^3} \mid n \geq 0\}$ is context-free.

   (b) Prove or disprove that the language $L = \{a^n b^m c^k \mid 3n + 2m = k\}$ is context-free.

9. **General Computability Results:** (*15 Points*) Do one of the following.

   (a) Prove that the cardinality of the set of all Turing machines is countable.

   (b) Prove that the cardinality of the power set of a set A is strictly greater than the cardinality of A.

   (c) Prove that the set of all real numbers, $\mathbb{R}$, is uncountable.

10. **General Computability Results:** (*15 Points*) Do one of the following.

    (a) Prove that there exists a function $f : \mathbb{N} \to \mathbb{N}$ that is not partial Turing computable.

    (b) Define the Halt (H) program as we did in class and then prove that it does not exist.