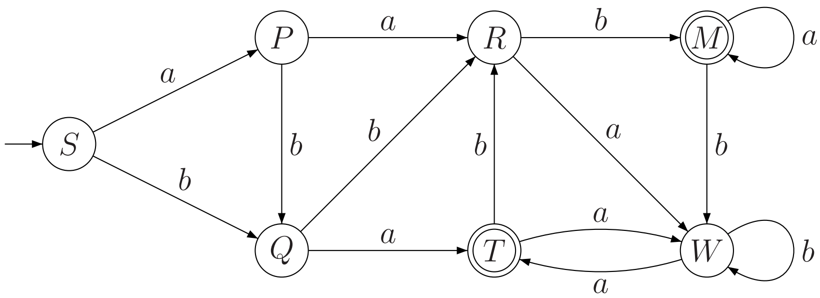
Spring 2012

Exam #1 Key

Math 362

1. (*20 Points*) Consider the following DFA, A.



   (a) Determine if the automaton accepts the following words. Display the sequence of states for each word.

      i. $baabbba$ — S Q T W W W W T (Accepted)

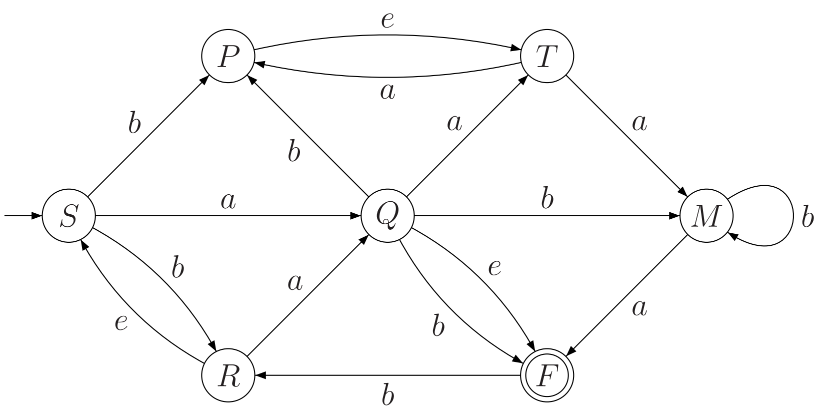
      ii. $aaaaa$ — S P R W T W (Not Accepted)

      iii. $aabaabb$ — S P R M M M W W (Not Accepted)

   (b) Is $L(bbaa(baba)^*) \subset L(A)$? Why or why not? — Yes, $bbaa$ lands in T and the sequence $baba$ from T lands back in T.

   (c) Is $\{b^n a^m \mid n, m > 0 \text{ and } n \text{ and } m \text{ are even}\} \subset L(A)$? Why or why not? — No, $bbbbaa$ is not accepted.

2. (*25 Points*) Consider the following NFA, A.



   (a) Determine if the automaton accepts the following words. If it does, display the sequence of states that drive the word to an acceptable state.

      i. $aababb$ — Not Accepted

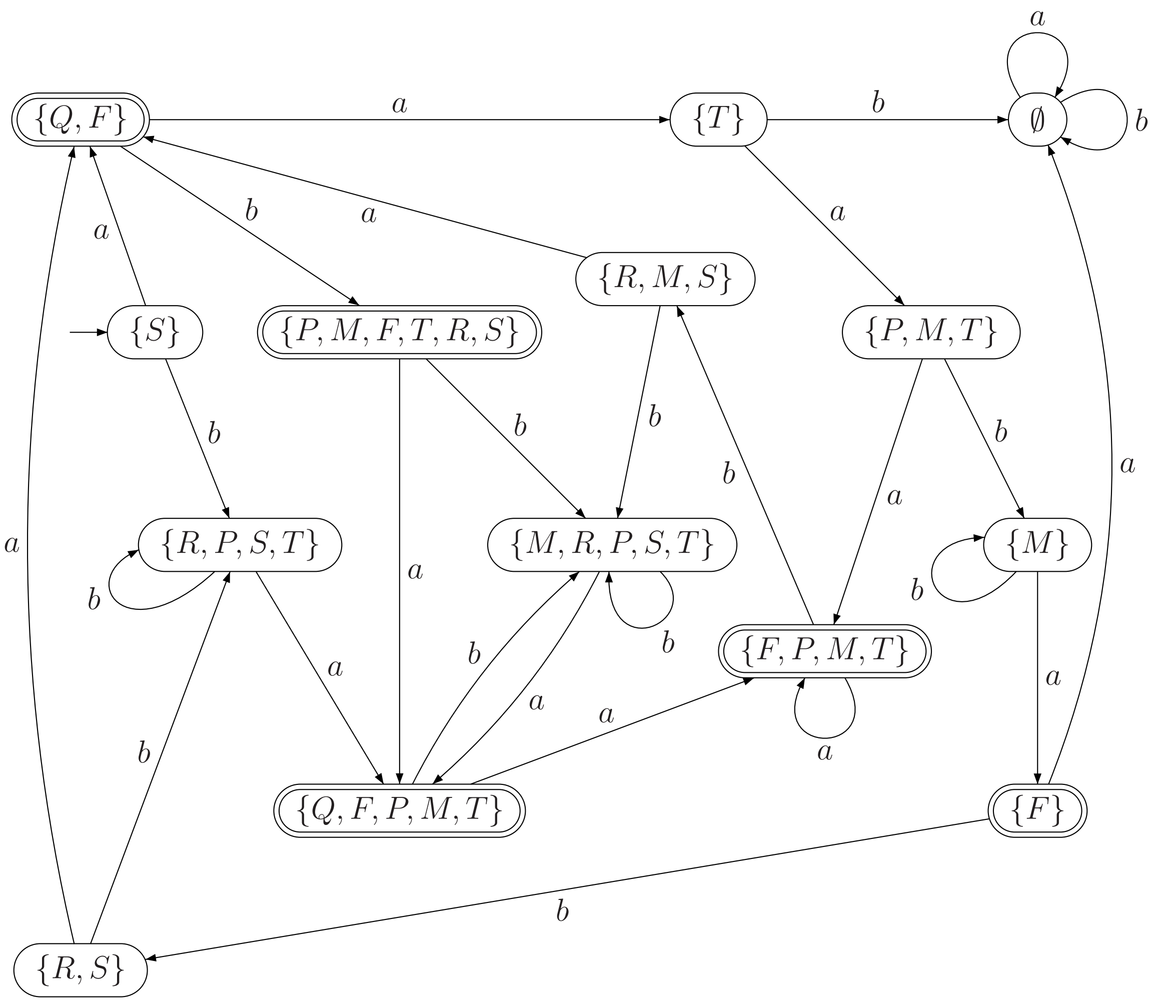
      ii. $babba$ — Accepted — S R Q F R Q F

      iii. $baba$ — Accepted — S R Q M F

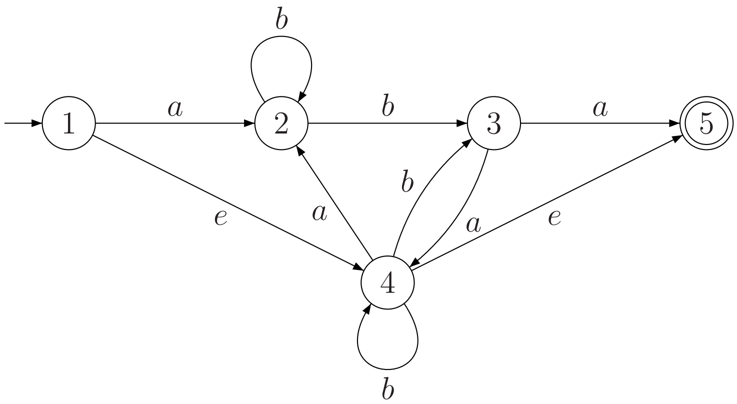
      iv. $aaaaa$ — Accepted — S Q T P T M F

   (b) Is $\{baaa(ba)^n \mid n > 0\} \subset L(A)$? Why or why not? — Yes, $baaa$ through S P T P T M F lands in F, then ba through R Q F lands back in F.

(c) Convert this NFA to a DFA.

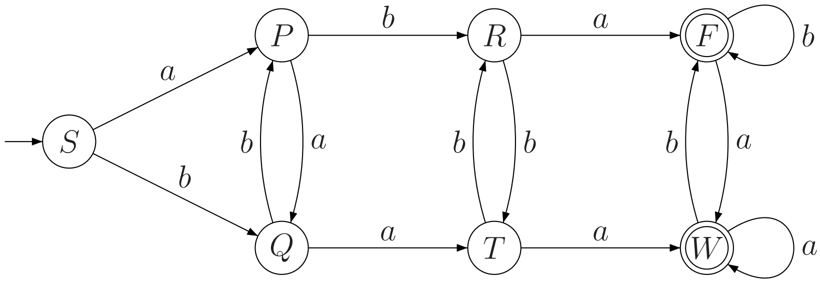


3. (*20 Points*) Convert the following NFA to a regular expression,



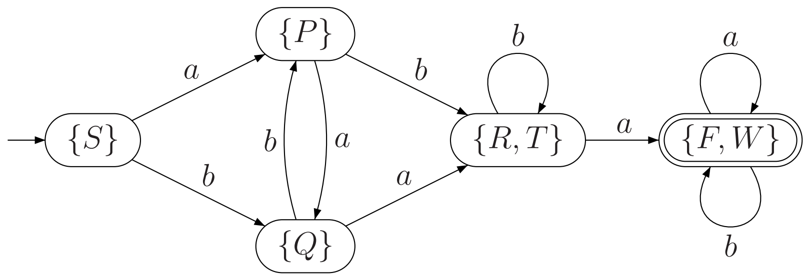
Solution: $ab^*ba \cup (e \cup ab^*ba)(b \cup (b \cup ab^*b)a)^*(e \cup (b \cup ab^*b)a)$.

4. (*20 Points*) Minimize the number of states for the the following DFA,



Solution: The equivalence class chart and the converted automaton are

0	1	2	3
SPQRT	SPQ	S	S
FW	RT	P	P
	FW	Q	Q
		RT	RT
		FW	FW



5. (*25 Points*) Prove that the language $L = \{a^t b^n \mid n > 0,$ and either $t = n$ or $t = 2n\}$ is not regular. Make sure you verify all statements completely.

Solution: Assume that L is a regular and let n be the value from the pumping lemma. Let $w = a^{2n}b^n$, then $w = xyz$ with $|xy| \le n$ and y non-empty. Thus, $xy = a^k$ for some $1 \le k \le n$ and so $y = a^p$ for some $1 \le p \le n$. But $xy^2z = a^{2n+p}b^n \notin L$, since $2n+p \ne 2n$ and $2n+p \ne n$, which contradicts L being regular.