Spring 2012 **Exam #2** Math 362

Name: _____

Write all of your responses on the extra exam paper provided. Turn in all work and this exam paper.

1. **Grammars and Parsing:** (*25 Points*) Consider the context-free grammar G with $\Sigma = \{a, b\}$ and rules
   (1) $S \longrightarrow AB$
   (2) $S \longrightarrow BA$
   (3) $A \longrightarrow aAb$
   (4) $B \longrightarrow bBa$
   (5) $B \longrightarrow e$
   (6) $A \longrightarrow e$

   (a) Construct a left-most derivation for the word *aaabbbba*. Put the rule number over the arrow at each step.

   (b) Construct the parse tree for your derivation of *aaabbbba*.

   (c) Is the language $L(G)$ ambiguous? If so, prove it.

   (d) Is $L(G)$ regular? If so write $L(G)$ as a regular expression and if not describe $L(G)$ in words.

2. **Push-Down Automata and Determinism:** (*25 Points*) Consider the following PDA, A. A has starting state s, accepts by empty stack, $\Sigma = \{a, b, c\}$ and $\Gamma = \{a, b, d\}$
   (1) $((s, e, e), (q, d))$
   (2) $((q, a, e), (q, b))$
   (3) $((q, b, e), (q, a))$
   (4) $((q, c, e), (t, e))$
   (5) $((t, a, a), (t, e))$
   (6) $((t, b, b), (t, e))$
   (7) $((t, e, d), (r, e))$
   (8) $((r, a, e), (r, e))$
   (9) $((r, b, e), (r, e))$

   (a) Use a transition/state/input/stack table (as done in class) to test the strings *aabcabbabab* and *aabcaab*.

   (b) Describe $L(A)$.

   (c) Is the automaton deterministic? Why or why not?

3. **True & False:** (*20 Points*) Mark each of the following as being either true or false.

   (a) _____ Any language that can be represented as the concatenation of a context-free language and a regular language can be accepted by a push-down automaton.

   (b) _____ The intersection of two context-free languages is context-free.

   (c) _____ The Kleene star of a context-free language is context-free.

   (d) _____ The union of a context-free language with a regular language is regular.

   (e) _____ The concatenation of a context-free language and a regular language is context-free.

   (f) _____ The complement of a deterministic context-free language is deterministic context-free.

   (g) _____ The complement of a context-free language can be represented as a finite union of context-free languages.

   (h) _____ In order for a language to be non-context-free the alphabet of that language must contain at least 3 distinct characters.

   (i) _____ The complement of a regular language is deterministic context-free.

   (j) _____ The intersection of a context-free language and a regular language is context-free.

4. **Context-Free Languages:** (*20 Points*) Show that the language $L = \{a^k b^{2k} c^{3k} \mid k = 0, 1, 2, \ldots\}$ is not context-free.

5. **Chomsky Normal Form:** (*20 Points*) Convert the following grammar to Chomsky Normal Form.

   (1) $S \longrightarrow AB$
   (2) $S \longrightarrow BA$
   (3) $A \longrightarrow aAb$
   (4) $B \longrightarrow bBa$
   (5) $B \longrightarrow e$
   (6) $A \longrightarrow e$