Spring 2012 **Exam #3** Math 362

Name: _____

Write all of your responses on the extra exam paper provided. Turn in all work and this exam paper.

1. **Short Answer: (***5 Points Each***):** Answer all of the following.
   (a) Define countably infinite.
   (b) Define a partial Turing computable function.
   (c) Define a decidable language.
   (d) Define a Turing enumerable language.
   (e) State the Church-Turing thesis.

2. **Determinism:** (*25 Points*) Show that the language $L = \{wcw^R \mid w \in \{a, b\}^*\}$ is deterministic context-free.

3. **Turing Machines:** (*25 Points Each*)
   (a) Write a complete set of transitions for a Turing Machine that semidecides the language $L = \{wcw^R \mid w \in \{a, b\}^*\}$.
   (b) Using the primitives R, L, $R_{\sqcup}$, $L_{\sqcup}$, $R_{\sqsubset}$, $L_{\sqsubset}$, $R_{\triangleright}$, $L_{\triangleright}$, $R_{\bar{\triangleright}}$, $L_{\bar{\triangleright}}$, R_0, L_0, R_1, L_1, $R_{\bar{0}}$, $L_{\bar{0}}$, $R_{\bar{1}}$, $L_{\bar{1}}$, R_a, L_a, $R_{\bar{a}}$, $L_{\bar{a}}$, R_b, L_b, $R_{\bar{b}}$, $L_{\bar{b}}$, Shl, Shr, A (add one), and S (subtract one) construct a Turing machine (in diagram form) that takes a word $w \in \{a, b\}^*$ and outputs the number of a's in binary form. For example, an input of $\triangleright\underline{b}bbbabbaaba$ produces $\triangleright\underline{1}00$.

   The Turing machine A (add one) will add one to a number string given that the read/write head is on the space after the number and it returns the read/write head to the space after the number before it halts.

   The Turing machine S (subtract one) will subtract one from a number string given that the read/write head is on the space after the number and it returns the read/write head to the space after the number before it halts.

4. **Infinity: (***10 Points***):** Prove that the cardinality of the power set of a set A, $\mathcal{P}(A)$, is strictly greater than the cardinality of A.