Fall 2013 **Exam #1** COSC 362

Name: _____

Write all of your responses on the exam paper or on the extra paper provided. Turn in all work and this exam paper.

1. (*10 Points*) Do one and only one of the following proofs,

   (a) Prove by induction that for $n \geq 4$, $2^n < n!$.

   (b) Prove by induction that for $\sum_{i=1}^{n} \frac{1}{i^2} \leq 2 - \frac{1}{n}$.

2. (*5 Points Each*) Answer each of the following questions on languages and grammars. For this exercise, $\Sigma = \{a, b\}$.

   (a) Give a grammar for the language $L_1$ of all odd-length palindromes.

   (b) Give a grammar for the language $L_2 = \{w \in \Sigma^* \mid n_a(w) = n_b(w) + 1\}$.

   (c) Give a grammar for the language $L_1 \cup L_2$.

   (d) Give a grammar for the language $L_1 L_2$.

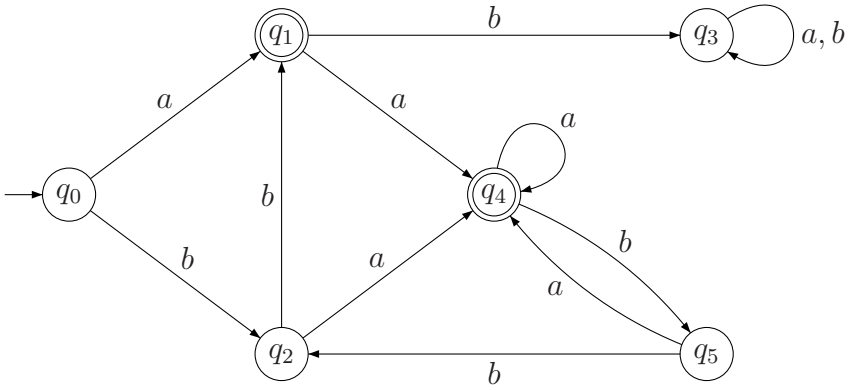
3. (*5 Points Each*) For each of the following languages, give a regular expression for that language. For this exercise, $\Sigma = \{a, b\}$.

   (a) $L = \{a^n b^m \mid (n + m) \text{ is even}\}$

   (b) $L$ is the language of all words with at most two occurrences of the substring $aa$.

   (c) $L = \{vwv \mid w \in \Sigma^* \text{ and } 1 \leq |v| \leq 2\}$

4. (*30 Points*) Consider the following DFA, $A$.




(a) Determine if the automaton accepts the following words. Display the sequence of states for each word.

i. *aabbaa*

ii. *bbaabbab*

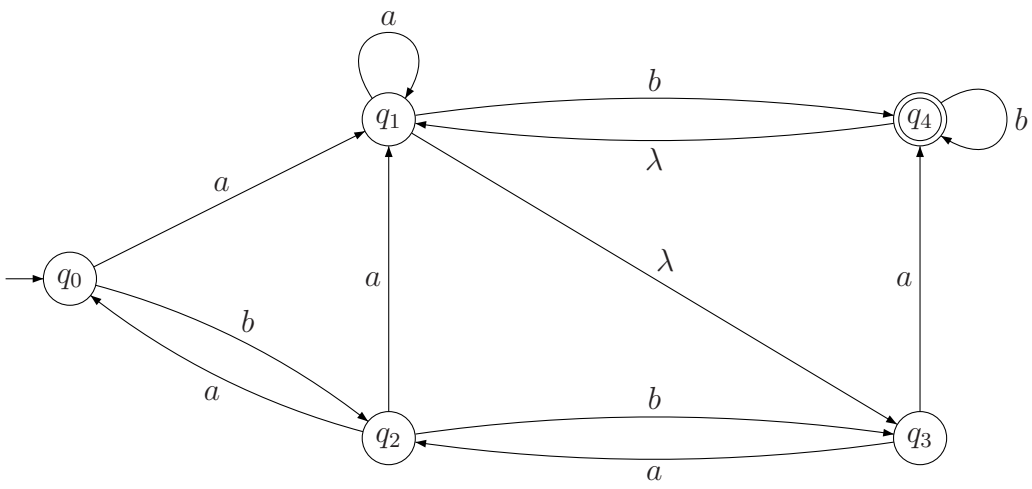
iii. *aababab*

(b) Is $L(aa(ba)^*) \subseteq L(A)$? Why or why not?

(c) For what values of $n$ and $m$ is $a^n b^m \in L(A)$?

(d) What is the smallest run of $b$'s that will guarantee that the word will not be accepted. That is, if $w = ub^n v$ for any $u, v \in \Sigma^*$, what is the smallest value of $n$ will guarantee that $w \notin L(A)$? Justify your answer.

5. (*35 Points*) Consider the following NFA, $A$.




(a) Determine if the automaton accepts the following words. If it does, display the sequence of states that drive the word to a final state.

i. *abab*

ii. *abbbaaba*

iii. *bbaab*

(b) Find a word of length 4 that is not accepted.

(c) Describe the language that is accepted by this automaton, $L(A)$.

(d) Convert this NFA to a DFA.