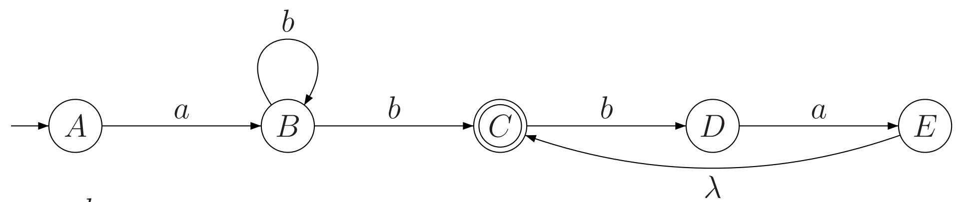
Exam #2 Key

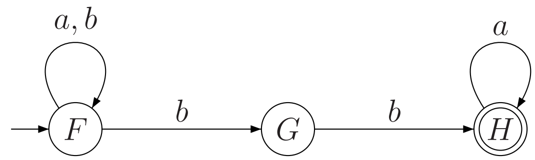
1. (*20 Points*) Let $L_1 = L(ab^*b(ba)^*)$ and $L_2 = L((a+b)^*bba^*)$, construct NFAs for each of the following languages, L_1, L_2, $\overline{L_1}$, and $L_1 \cup L_2$.

Solution:

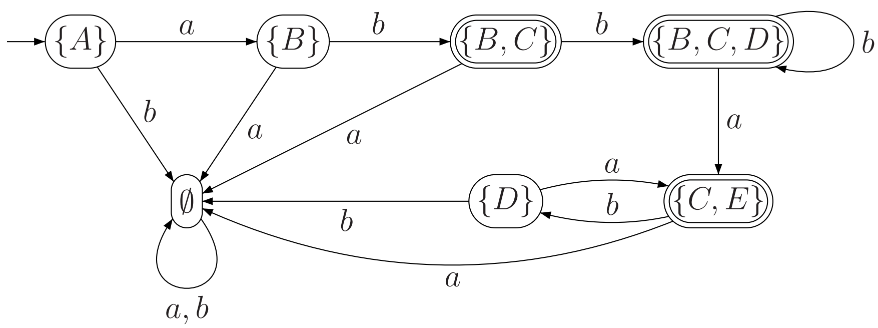
L_1



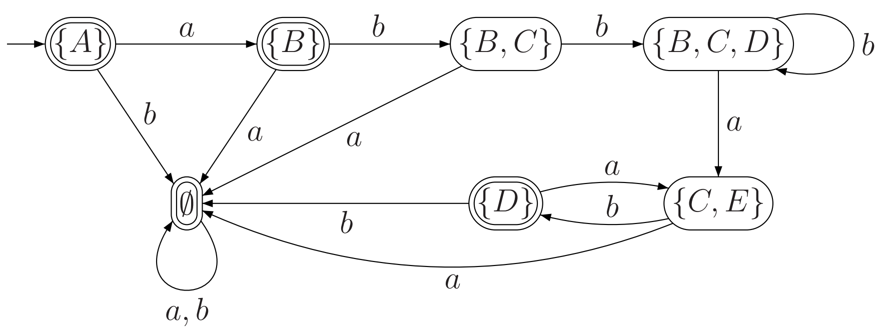
L_2



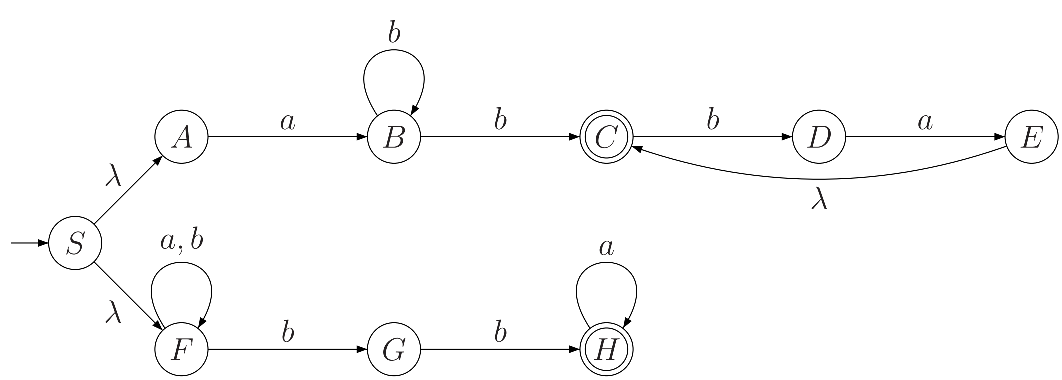
L_1: DFA



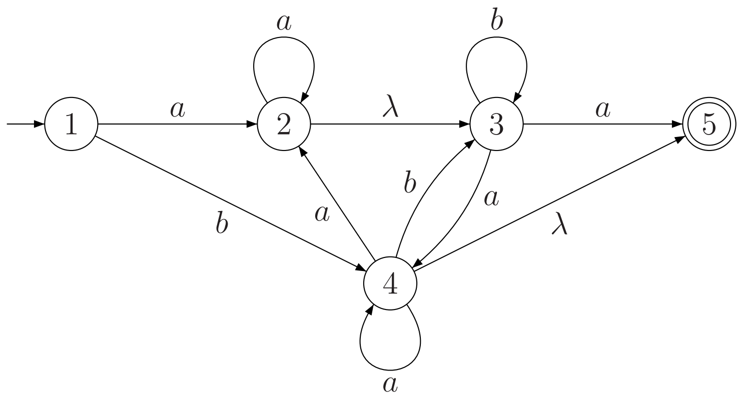
$\overline{L_1}$



$L_1 \cup L_2$

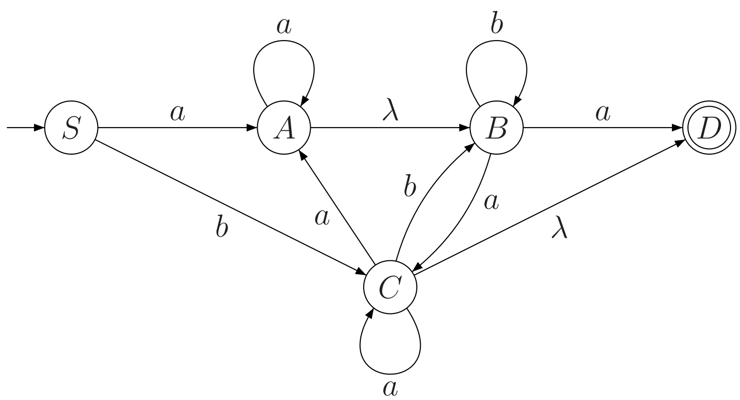


2. (*20 Points*) Convert the following NFA to a regular expression.



Solution: $aa^*b^*a + (b + aa^*b^*a)(a + (b + aa^*)b^*a)(\lambda + (b + aa^*)b^*a)$

3. (*10 Points*) Construct a right linear grammar for the following NFA.

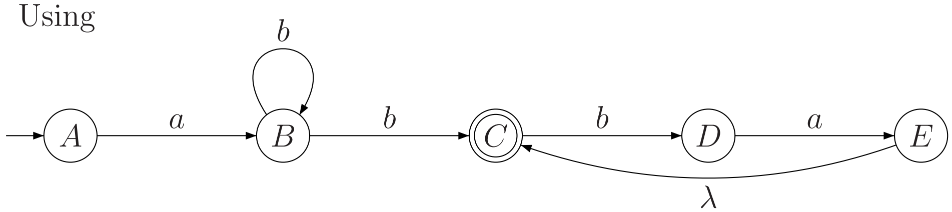


Solution:

$S \to aA$	$A \to aA$	$B \to bB$	$B \to aD$	$C \to D$	$C \to aA$
$S \to bC$	$A \to B$	$B \to aC$	$C \to aC$	$C \to bB$	$D \to \lambda$

4. (*10 Points*) Construct a regular grammar for the language $L(ab^*b(ba)^*)$.

Solution: Using



$A \to aB$	$B \to bC$	$D \to aE$	$C \to \lambda$
$B \to bB$	$C \to bD$	$E \to C$	

5. (*10 Points Each*) For each of the following languages, determine if it is regular or not regular, justify your answer with a proof.

   (a) $L_1 = \{a^n b^q a^k \mid n = q \text{ or } q \neq k\}$

   Solution: L_1 is not regular.

   Assume that L_1 is regular and let m be the value from the pumping lemma. Let $w = a^m b^m a^m$, then since $|w| = 3m \geq m$ we can write $w = xyz$ with $|xy| \leq m$ and y non-empty. Thus, $xy = a^k$ for some $1 \leq k \leq m$ and so $y = a^p$ for some $1 \leq p \leq m$. But $xy^2z = a^{m+p}b^m a^m \notin L_1$, since $n = m + p \neq m = q$ and $q = m = k$, which contradicts L_1 being regular.

   (b) $L_2 = \{a^n \mid n = 2^k \text{ for some } k \geq 0\}$

   Solution: L_2 is not regular.

   Assume that L_2 is regular and let m be the value from the pumping lemma. Let $w = a^{2^m}$, then since $|w| = 2^m \geq m$ we can write $w = xyz$ with $|xy| \leq m$ and y non-empty. Thus, $xy = a^k$ for some $1 \leq k \leq m$ and so $y = a^p$ for some $1 \leq p \leq m$. But $xy^2z = a^{2^m+p} \notin L_2$, since $p \leq m < 2^m$ we have $2^m + p < 2^m + 2^m = 2 \cdot 2^m = 2^{m+1}$, which is the next power of 2, this contradicts L_2 being regular.

   (c) $L_3 = \{a^n b^q \mid n + q \geq 2\}$

   Solution: L_3 is regular since $L_3 = L(aaa^*b^* + aa^*bb^* + a^*bbb^*)$

   (d) $L_4 = \{a^n b^q c^k \mid n + k \leq 7 \text{ and } n < q < k\}$

   Solution: L_4 is regular since L_4 is finite and all finite languages are regular.

   (e) $L_5 = \{w \in (a + b)^* \mid n_a(w) = 2n_b(w)\}$

   Solution: L_5 is not regular.

   Assume that L_5 is regular and let m be the value from the pumping lemma. Let $w = a^{2m}b^m$, then since $|w| = 3m \geq m$ we can write $w = xyz$ with $|xy| \leq m$ and y non-empty. Thus, $xy = a^k$ for some $1 \leq k \leq m$ and so $y = a^p$ for some $1 \leq p \leq m$. But $xy^2z = a^{2m+p}b^m \notin L_5$, since $p > 0$ we have $2m + p > 2m$, this contradicts L_5 being regular.