Spring 2014

Final Exam

COSC 362

Name: ____

Write all of your responses on the exam paper or on the extra paper provided. Turn in all work and this exam paper.

1. **Definitions & Short Answer:** (*30 Points*)

   (a) Discuss the primary differences between Finite Automata, Pushdown Automata, and Turing Machines as far as models of computation are concerned.

   (b) What is a leftmost derivation?

   (c) Define a regular grammar.

   (d) What does it mean for a grammar to be ambiguous?

(e) State Turing's Thesis.

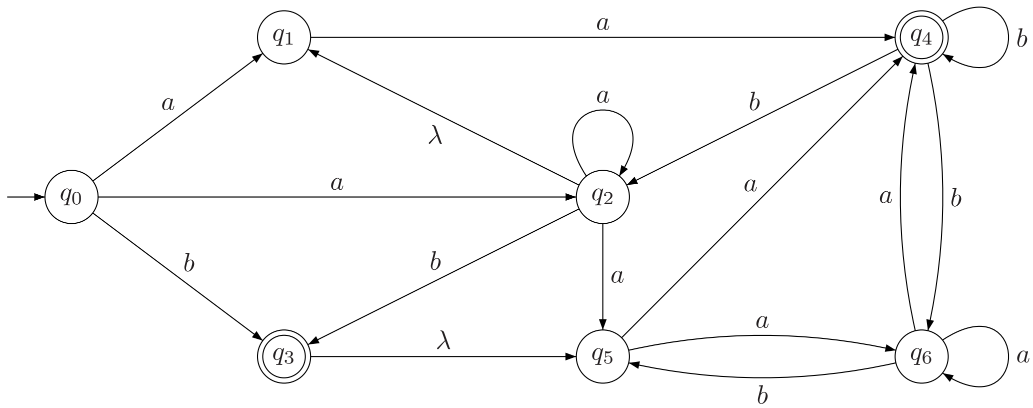
(f) Define an algorithm.

(g) State Rice's Theorem and briefly discuss its consequences.

(h) State the Halting Problem and briefly discuss its consequences.

2. **Finite Automata:** (*15 Points*) Construct a DFA that will accept the language of all words over $\Sigma = \{a, b\}$ that have at most two runs of a's of length three, and no runs of length greater than three.

3. **Finite Automata:** (*15 Points*) Consider the following NFA, A.



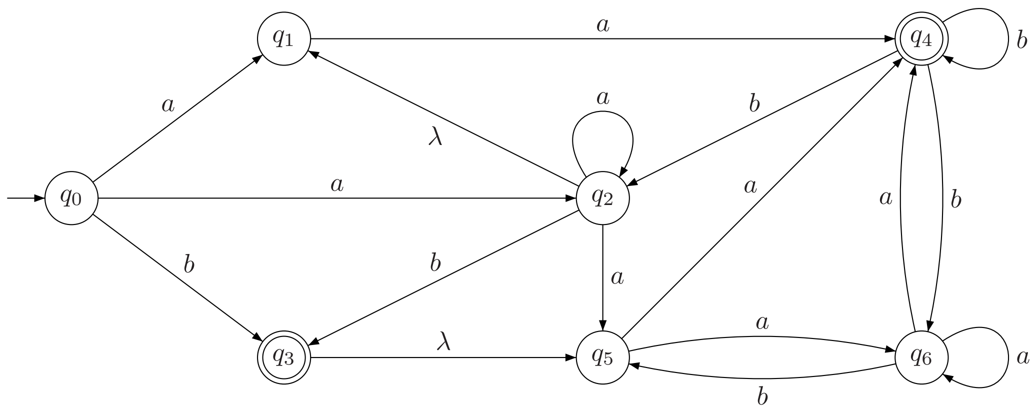
(a) Is *aaabbab* acceptable or not acceptable? If it is acceptable display a sequence of states for the word that end in a favorable state. If it is not acceptable give a *short* explanation why.

(b) Is *aaabaaba* acceptable or not acceptable? If it is acceptable display a sequence of states for the word that end in a favorable state. If it is not acceptable give a *short* explanation why.

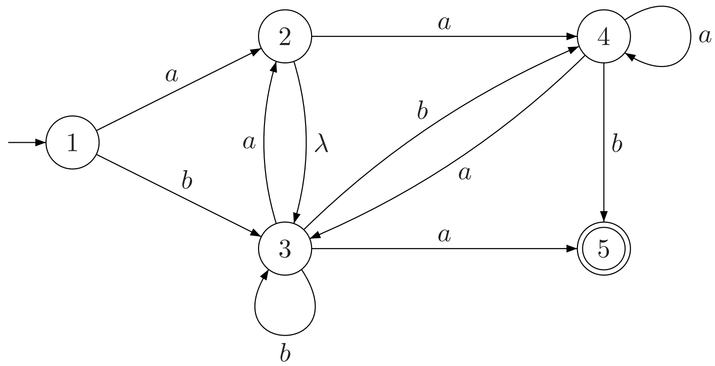
(c) Is *bbaababba* acceptable or not acceptable? If it is acceptable display a sequence of states for the word that end in a favorable state. If it is not acceptable give a *short* explanation why.

(d) Is $\{a^n b^n \mid n \geq 1\} \subset L(A)$? Prove or disprove your answer.

4. **Finite Automata:** (*15 Points*) Convert this NFA to a DFA and then write a regular expression for the language accepted by the automaton.



5. **Finite Automata:** (*15 Points*) Convert the following NFA to a regular expression.



6. **Context-Free Languages, Grammars & Push-Down Automata:** (*20 Points*) Construct a push-down automaton that accepts the language $L = \{a^n b^{n+m} c^m \mid n \geq 1, m \geq 1\}$.

7. **Context-Free Languages, Grammars & Push-Down Automata:** (*30 Points*) This exercise deals with the language $L = \{vwv^R \mid v \in \{a,b\}^*, w \in \{c,d\}^*\}$.

   (a) Construct a context-free grammar for the language L.

   (b) Convert your grammar from 7a to Chomsky Normal Form.

(c) Construct a push-down automaton that accepts the language $L = \{vwv^R \mid v \in \{a, b\}^*, w \in \{c, d\}^*\}$.

8. **Turning Machines:** (*25 Points*) Construct a standard Turing Machine by displaying the set of transitions for the Turing Machine that will compute the function $f(w) = w^R$ where $w \in \{a, b\}^+$.

9. **Turning Machines:** (*25 Points*) Use the primitives R, L, R_a, L_a, R_b, L_b, $R_\square$, $L_\square$, R_0, L_0, R_1, L_1, $R_{\overline{a}}$, $L_{\overline{a}}$, $R_{\overline{b}}$, $L_{\overline{b}}$, $R_{\overline{\square}}$, $L_{\overline{\square}}$, $R_{\overline{0}}$, $L_{\overline{0}}$, $R_{\overline{1}}$, $L_{\overline{1}}$, a, b, 0, 1, $\square$, A, S, Shl, Shr, N_L, N_R, W_E and W_B, and the tape alphabet of $\{a, b, 0, 1, \square\}$ where,

   A — Adds one in binary, the read/write head begins and ends on the leftmost digit. So applying it to $\underline{1}00101$ produces $\underline{1}00110$. Also the number grows to the left, so $\square\underline{1}11$ produces $\underline{1}000$.

   S — Subtracts one in binary, the read/write head begins and ends on the leftmost digit. So applying it to $\underline{1}00110$ produces $\underline{1}00101$. Also the number shrinks on the left, so $\underline{1}000$ produces $\square\underline{1}11$.

   Shl — Shifts a word one space to the left. So $\square\underline{a}ba$ produces $\underline{a}ba\square$.

   Shr — Shifts a word one space to the right. So $\underline{a}ba\square$ produces $\square\underline{a}ba$.

   N_L — Moves the read/write head to the beginning of the next word to the left.

   N_R — Moves the read/write head to the beginning of the next word to the right.

   W_E — Moves the read/write head to the end of the word. If the read/write head is on a space the head does not move.

   W_B — Moves the read/write head to the beginning of the word. If the read/write head is on a space the head does not move.

   Construct a Turing machine (in diagram form) that will take an input of a single word from $\{a, b\}^*$ and write the number of characters in binary on front of the word. The original word is not altered by the computation. For example, if the input tape is $\square\underline{a}bbbababbaa\square$ the Turing machine produces $\square 1011 \square\underline{a}bbbababbaa\square$. You may use other tape symbols if you would like, in this case the R_x, $R_{\overline{x}}$, L_x and $L_{\overline{x}}$ machines work as usual.

10. **Membership:** (*10 Points*) Do one and only one of the following.

(a) Prove that the language $L = \{w \mid n_a(w)/n_b(w) = n_c(w)\}$ is not context-free.

(b) Prove that the language $L = \{a^n b^m \mid n \leq m \leq 2n\}$ is not regular.

11. **Computability:** (*10 Points*) Do one and only one of the following.

   (a) Prove that there exists a function $f : \mathbb{N} \to \mathbb{N}$ that is not partial Turing computable.

   (b) Define the Halt (H) program as we did in class and then prove that it does not exist.