Name: _____

Write all of your responses on the extra paper provided. Hand in this exam paper along with your solutions, please place your name on the top of each page.

1. (10 points) Use induction to prove that for all $n \geq 1$,

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

2. (25 Points) Answer each of the following questions on languages and grammars. For this exercise, $\Sigma = \{a, b\}$.

   (a) Find a grammar that generates the language of all palindromes, that is,

   $$L = \{w \in \Sigma^* \mid w = w^R\}$$

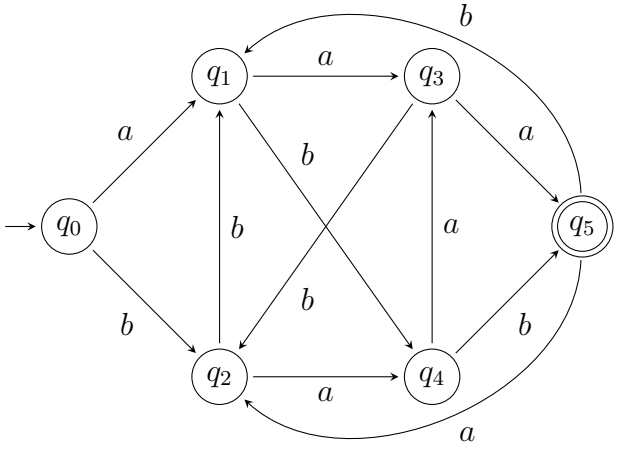
   (b) Find a grammar that generates the language,

   $$L = \{a^n b^m \mid n \geq 0 \text{ and } m \geq 0\}$$

   (c) Find a grammar that generates the union of the above two languages, specifically,

   $$L = \{w \in \Sigma^* \mid w = w^R\} \cup \{a^n b^m \mid n \geq 0 \text{ and } m \geq 0\}$$

3. (20 Points) Consider the following DFA, M



   (a) Determine if the automaton accepts the following words. Display the sequence of states for each word.

      i. *abbaba*
      ii. *bbbbbb*
      iii. *ababab*
      iv. *bbbabbb*

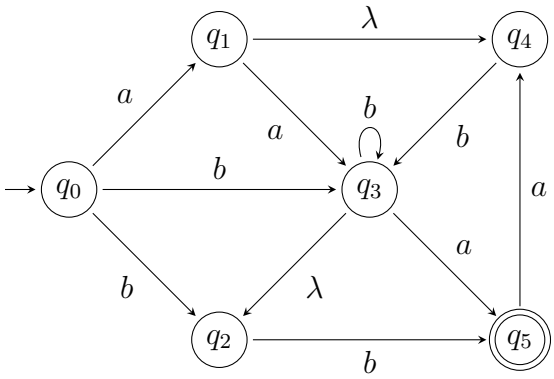
(b) If $a^n \in L(M)$ then what are all of the possible values of n.

(c) If $b^n \in L(M)$ then what are all of the possible values of n.

4. (10 Points) Construct a DFA, with $\Sigma = \{a, b\}$, that accepts the language

$$L = \{ab^n a^m \mid n \geq 2 \text{ and } m \geq 3\}$$

5. (25 Points) Consider the following NFA, M



(a) Determine if the automaton accepts the following words. Display the sequence of states for each accepted word and if the word is not accepted, give a short explanation of why.

i. *abbaba*

ii. *bbbbbbb*

iii. *ababab*

iv. *bbbabbb*

(b) If $a^n \in L(M)$ then what are all of the possible values of n.

(c) If $b^n \in L(M)$ then what are all of the possible values of n.

(d) Convert the NFA to a DFA

6. (20 Points) Minimize the number of states of the following DFA, and display the minimal state automaton. Make sure you show all steps in the conversion algorithm.