Name: _____

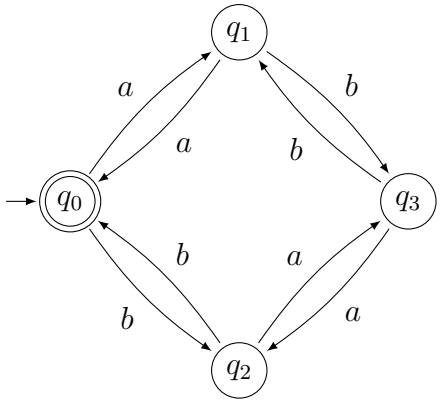
Write all of your responses on the extra paper provided. Hand in this exam paper along with your solutions, please place your name on the top of each page.

1. (5 points Each) Find regular expressions for the following languages, $\Sigma = \{a, b\}$.
   (a) $L = \{a^m b^n w \mid w \in \Sigma^*, m \geq 4, n \leq 3\}$
   (b) $L = \{vwv \mid w \in \Sigma^*, |v| = 3\}$
   (c) $L = \{w \mid n_a(w) \mod 3 = 1\}$
   (d) $L = \{w \mid w \in \Sigma^*, w \text{ contains exactly one pair of consecutive } a\text{'s}\}$

2. (10 points Each) Prove the following,
   (a) Given a set of n regular languages $\{L_1, L_2, L_3, \ldots, L_n\}$, show that the union of these is a regular language, that is, show that $L = L_1 \cup L_2 \cup L_3 \cup \cdots \cup L_n$ is regular.
   (b) Given two regular languages L_1 and L_2 show that the reversal difference, R, is regular. The reversal difference is defined to be

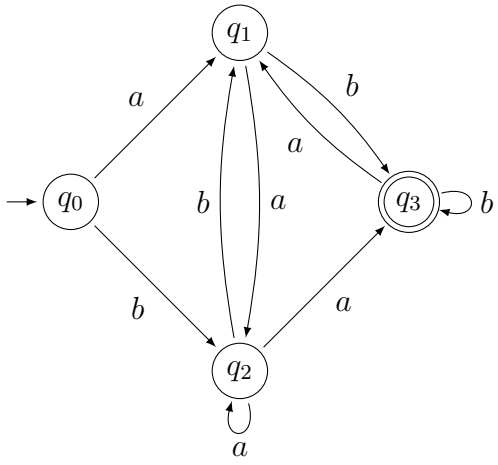
$$R = \{w \in \Sigma^* \mid w \in L_1 \text{ and } w^R \notin L_2\}$$

3. (20 points) The following finite automaton is one possible automaton for the language $L = \{w \in \{a, b\}^* \mid n_a(w) \text{ and } n_b(w) \text{ are both even.}\}$.



   (a) Using the algorithm discussed in class, convert this automaton to a regular grammar.
   (b) Use the grammar to derive the word *aababa*.

4. (20 points) Consider the following automaton. Using the algorithm discussed in class, convert this automaton to a regular expression. Show all of your steps in the conversion process.



5. (10 Points Each) For each of the following languages, determine if it is regular or not regular, justify your answers.

   (a) $L = \{a^n b^q a^k \mid n = q \text{ or } k \neq q\}$

   (b) $L = \{a^n b^k \mid n < 2k\}$

6. (10 Points) Prove or disprove the following statement: If L_1 and L_2 are nonregular languages, then $L_1 \cup L_2$ is also a nonregular language.