Name: _____

Write all of your responses on the extra paper provided. Hand in this exam paper along with your solutions, please place your name on the top of each page.

1. (*10 Points Each*) Construct context-free grammars for the following languages,

   (a) $L = \{a^n b^{n+m} c^m \mid n \geq 0 \text{ and } m \geq 0\}$

   (b) $L = \{wcw^R \mid w \in \{a, b\}^*\}$

2. (*5 Points Each*) Consider the following context free grammar, G,

$$\begin{aligned} S &\rightarrow aABb|aBAb|abAB \\ A &\rightarrow aBA|BAb|bB|b \\ B &\rightarrow bAB|ABa|aA|a \end{aligned}$$

   (a) Find a leftmost derivation for the word *abbaab*.

   (b) Construct the derivation tree for the derivation in 2a.

   (c) Show that this grammar is ambiguous.

3. (*10 Points Each*) Create nondeterministic push-down automata that accept the following languages,

   (a) $L = L(ba^*abb) \cup L(a^*ba^*)$

   (b) $L = \{a^n b^{n+m} c^m \mid n \geq 0 \text{ and } m \geq 0\}$

4. (*10 Points Each*) Consider the following context free grammar, G,

$$\begin{aligned} S &\rightarrow aAbb|bBaa|abAB \\ A &\rightarrow abBA|ab|a \\ B &\rightarrow bBBb|aA|b \end{aligned}$$

   (a) Convert this grammar to Greibach Normal Form.

   (b) Using the Greibach Normal Form, convert the grammar into a nondeterministic push-down automaton.

5. (*15 Points*) Construct a deterministic push-down automaton that accepts the language,

$$L = \{wcw^R \mid w \in \{a, b\}^*\}$$

6. (*20 Points*) Prove that the language $L = \{w \in \{a, b, c\}^* \mid n_a(w) < n_b(w) < n_c(w)\}$ is not context-free.