Name: ____

Write all of your responses on these exam pages. If you need extra space please use the backs of the pages. Show all your work, answers without supporting justification will not receive credit. Each exercise is worth 10 points. Keep your answers in exact form. **No calculation devices allowed.**

1. Solve the following equation, find all solutions both real and complex.

$$2x^2 + 12x - 12 = 0$$

Solution: $2x^2 + 12x - 12 = 0$ is the same as $x^2 + 6x - 6 = 0$. Using the quadratic formula we have,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-6 \pm \sqrt{6^2 - 4 \cdot 1 \cdot (-6)}}{2 \cdot 1} = \frac{-6 \pm \sqrt{60}}{2} = \frac{-6 \pm 2\sqrt{15}}{2} = -3 \pm \sqrt{15}$$

2. Solve the following equation, find all solutions both real and complex.

$$x^4 - 3x^2 - 10 = 0$$

Solution: Let $t = x^2$ then $0 = x^4 - 3x^2 - 10 = t^2 - 3t - 10 = (t-5)(t+2)$. So either $t - 5 = 0$ or $t + 2 = 0$, that is, either $x^2 - 5 = 0$ or $x^2 + 2 = 0$. The first equation gives the solutions of $x = \pm\sqrt{5}$ and the second gives the solutions $x = \pm\sqrt{2}i$.

3. Solve the following equation.

$$\sqrt{3x+1} - \sqrt{x+4} = 1$$

Solution:

$$\begin{aligned}
\sqrt{3x+1} - \sqrt{x+4} &= 1 \\
\sqrt{3x+1} &= 1 + \sqrt{x+4} \\
(\sqrt{3x+1})^2 &= (1 + \sqrt{x+4})^2 \\
3x + 1 &= 1 + 2\sqrt{x+4} + x + 4 \\
2x - 4 &= 2\sqrt{x+4} \\
x - 2 &= \sqrt{x+4} \\
(x-2)^2 &= (\sqrt{x+4})^2 \\
x^2 - 4x + 4 &= x + 4 \\
x^2 - 5x &= 0 \\
x(x-5) &= 0
\end{aligned}$$

This gives the solutions of $x = 0$ and $x = 5$. Checking our solutions we see that $x = 5$ is a valid solution but $x = 0$ is not since it gives us -1 on the left hand side.

4. Solve the equation by completing the square. State all solutions, both real and complex.

$$3x^2 - 12x + 3 = 0$$

Solution:

$$\begin{aligned} 3x^2 - 12x + 3 &= 0 \\ x^2 - 4x + 1 &= 0 \\ x^2 - 4x &= -1 \\ x^2 - 4x + 4 &= -1 + 4 \\ (x-2)^2 &= 3 \\ x - 2 &= \pm\sqrt{3} \\ x &= 2 \pm \sqrt{3} \end{aligned}$$

5. Solve the following inequality, and express the solution in terms of intervals.

$$(3x-1)(10x+4) \geq (6x-5)(5x-7)$$

Solution:

$$\begin{aligned} (3x-1)(10x+4) &\geq (6x-5)(5x-7) \\ 30x^2 - 10x + 12x - 4 &\geq 30x^2 - 25x - 42x + 35 \\ 69x - 39 &\geq 0 \\ x &\geq \frac{39}{69} = \frac{13}{23} \end{aligned}$$

So the solution is $\left[\frac{13}{23}, \infty\right)$.

6. Solve the following inequality and express the solution in terms of intervals.

$$|4x + 7| > 21$$

Solution: $|4x+7| > 21$ is equivalent to $4x + 7 > 21$ or $4x + 7 < -21$. So $4x > 14$ or $4x < -28$, and hence $x > \frac{7}{2}$ or $x < -7$. So the solution is $(-\infty, -7) \cup \left(\frac{7}{2}, \infty\right)$.

7. Solve the following for h.

$$c = \sqrt{4h(2R-h)}$$

Solution:

$$\begin{aligned} c &= \sqrt{4h(2R-h)} \\ c^2 &= 4h(2R-h) \\ c^2 &= 8hR - 4h^2 \\ 4h^2 - 8Rh + c^2 &= 0 \\ h &= \frac{8R \pm \sqrt{64R^2 - 16c^2}}{8} \\ h &= \frac{2R \pm \sqrt{4R^2 - c^2}}{2} \end{aligned}$$

8. Express in the form $a + bi$, where a and b are real numbers.

$$\frac{2-5i}{7+i}$$

Solution:

$$\frac{2-5i}{7+i} = \frac{2-5i}{7+i} \cdot \frac{7-i}{7-i} = \frac{14-35i-2i+5i^2}{50} = \frac{9-37i}{50} = \frac{9}{50} - \frac{37}{50}i$$

9. Find an equation of the circle that has endpoints of a diameter $A = (8, 10)$ and $B = (0, 4)$.

Solution: The midpoint will be the center, which is $(4, 7)$. The radius will be half the distance between A and B, which is $r = \frac{1}{2}\sqrt{8^2 + 6^2} = \frac{1}{2}\sqrt{100} = 5$. So the equation of the circle is $(x-4)^2 + (y-7)^2 = 25$.

10. Find the equation of a line in slope-intercept form that passes through the point $A = (4, 5)$ and is perpendicular to the line $4x + 10y = 3$.

Solution: The line $4x + 10y = 3$ can be put into slope-intercept form of $y = -\frac{2}{5}x + \frac{3}{10}$. So the slope of the line we want is $m = \frac{5}{2}$. The line we want in point-slope form is $y - 5 = \frac{5}{2}(x - 4)$, solving for y and simplifying gives us $y = \frac{5}{2}x - 5$.