Name: ____

Write all of your responses on these exam pages. If you need extra space please use the backs of the pages. Show all your work, answers without supporting justification will not receive credit. Each exercise is worth 10 points. Keep your answers in exact form.

1. Solve the following equation, find all solutions both real and complex.

$$3x^2 + 12x - 9 = 0$$

Solution: $3x^2 + 12x - 9 = 0$ is the same as $x^2 + 4x - 3 = 0$. Using the quadratic formula we have,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-4 \pm \sqrt{4^2 - 4 \cdot 1 \cdot (-3)}}{2 \cdot 1} = \frac{-4 \pm \sqrt{28}}{2} = \frac{-4 \pm 2\sqrt{7}}{2} = -2 \pm \sqrt{7}$$

2. Solve the following equation, find all solutions both real and complex.

$$x^4 - 5x^2 - 6 = 0$$

Solution: Let $t = x^2$ then $0 = x^4 - 5x^2 - 6 = t^2 - 5t - 6 = (t-6)(t+1)$. So either $t - 6 = 0$ or $t + 1 = 0$, that is, either $x^2 - 6 = 0$ or $x^2 + 1 = 0$. The first equation gives the solutions of $x = \pm\sqrt{6}$ and the second gives the solutions $x = \pm i$.

3. Solve the following equation.

$$\sqrt{3x+6} - \sqrt{x-1} = 3$$

Solution:

$$\begin{aligned}
\sqrt{3x+6} - \sqrt{x-1} &= 3 \\
\sqrt{3x+6} &= 3 + \sqrt{x-1} \\
(\sqrt{3x+6})^2 &= (3 + \sqrt{x-1})^2 \\
3x + 6 &= 9 + 6\sqrt{x-1} + x - 1 \\
2x - 2 &= 6\sqrt{x-1} \\
x - 1 &= 3\sqrt{x-1} \\
(x-1)^2 &= (3\sqrt{x-1})^2 \\
x^2 - 2x + 1 &= 9(x-1) \\
x^2 - 2x + 1 &= 9x - 9 \\
x^2 - 11x + 10 &= 0 \\
(x-10)(x-1) &= 0
\end{aligned}$$

This gives the solutions of $x = 10$ and $x = 1$, both are valid.

4. Solve the equation by completing the square. State all solutions, both real and complex.

$$2x^2 - 24x + 14 = 0$$

Solution:

$$\begin{aligned}
2x^2 - 24x + 14 &= 0 \\
2x^2 - 24x &= -14 \\
x^2 - 12x &= -7 \\
x^2 - 12x + 36 &= -7 + 36 \\
(x-6)^2 &= 29 \\
x - 6 &= \pm\sqrt{29} \\
x &= 6 \pm \sqrt{29}
\end{aligned}$$

5. Solve the following inequality, and express the solution in terms of intervals.

$$\frac{x^2(3-x)}{x+2} \leq 0$$

Solution: Using a sign diagram the intervals where the expression is negative would be $(-\infty, -2)$ and $(3, \infty)$. Since we are including the possibility of equaling 0 we also get $x = 3$ and $x = 0$. Putting all this together we get $(-\infty, -2) \cup [3, \infty) \cup \{0\}$.

6. Solve the following inequality and express the solution in terms of intervals.

$$|5x + 2| < 15$$

Solution: $|5x+2| < 15$ is equivalent to $-15 < 5x + 2 < 15$. So this is $-17 < 5x < 13$ and hence $-\frac{17}{5} < x < \frac{13}{5}$. So the solution is $\left(-\frac{17}{5}, \frac{13}{5}\right)$.

7. Solve the following for h.

$$S = \pi r\sqrt{r^2 + h^2}$$

Solution:

$$\begin{aligned}
S &= \pi r\sqrt{r^2 + h^2} \\
\frac{S}{\pi r} &= \sqrt{r^2 + h^2} \\
\left(\frac{S}{\pi r}\right)^2 &= r^2 + h^2 \\
\left(\frac{S}{\pi r}\right)^2 - r^2 &= h^2 \\
h &= \pm\sqrt{\left(\frac{S}{\pi r}\right)^2 - r^2}
\end{aligned}$$

8. Express in the form $a + bi$, where a and b are real numbers.

$$\frac{5-2i}{2+3i}$$

Solution:

$$\frac{5-2i}{2+3i} = \frac{5-2i}{2+3i} \cdot \frac{2-3i}{2-3i} = \frac{(5-2i)(2-3i)}{13} = \frac{10-4i-15i+6i^2}{13} = \frac{4-19i}{13} = \frac{4}{13} - \frac{19}{13}i$$

9. Find the center and radius of the following circle.

$$x^2 + y^2 - 4x + 10y + 26 = 0$$

Solution:

$$\begin{aligned}
x^2 + y^2 - 4x + 10y + 26 &= 0 \\
x^2 + y^2 - 4x + 10y &= -26 \\
x^2 - 4x + y^2 + 10y &= -26 \\
x^2 - 4x + 4 + y^2 + 10y + 25 &= -26 + 25 + 4 \\
(x-2)^2 + (y+5)^2 &= 3
\end{aligned}$$

So the center is at $(2, -5)$ and the radius is $\sqrt{3}$.

10. Find the equation of a line in slope-intercept form that passes through the point $A = (2, -1)$ and is parallel to the line $6x - 4y = 7$.

Solution: The line $6x - 4y = 7$ can be put into slope-intercept form of $y = \frac{3}{2}x - \frac{7}{4}$. So the slope of the line we want is $m = \frac{3}{2}$. The line we want in point-slope form is $y + 1 = \frac{3}{2}(x - 2)$, solving for y and simplifying gives us $y = \frac{3}{2}x - 4$.