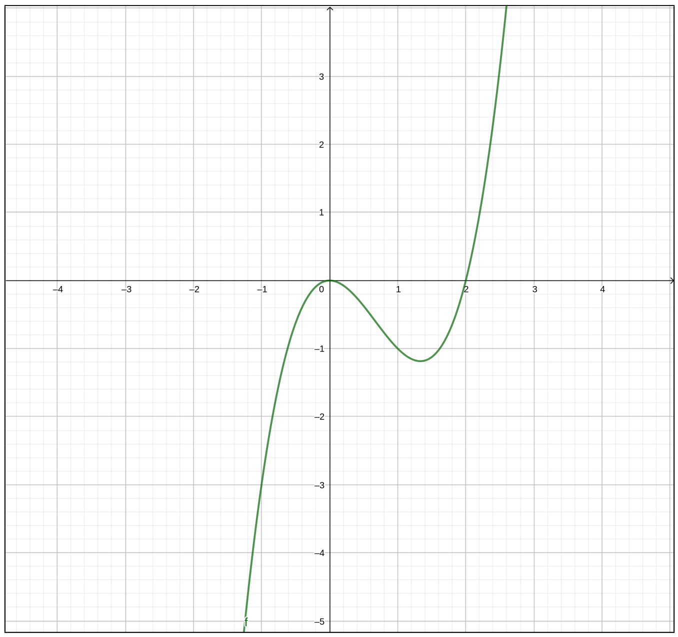
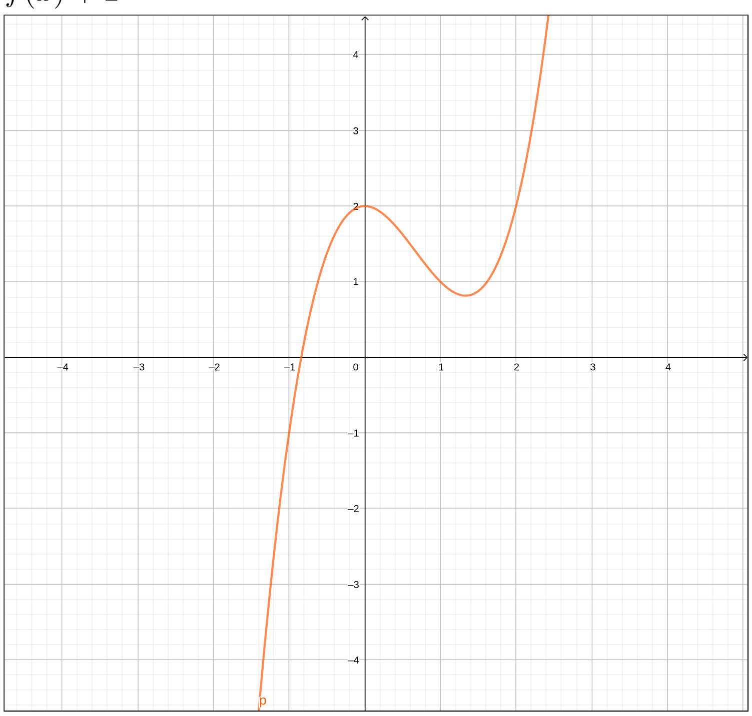
Name: ______________________________

Write all of your responses on these exam pages. If you need extra space please use the backs of the pages. Show all your work, answers without supporting justification will not receive credit. Keep your answers in exact form. **No calculation devices allowed.**

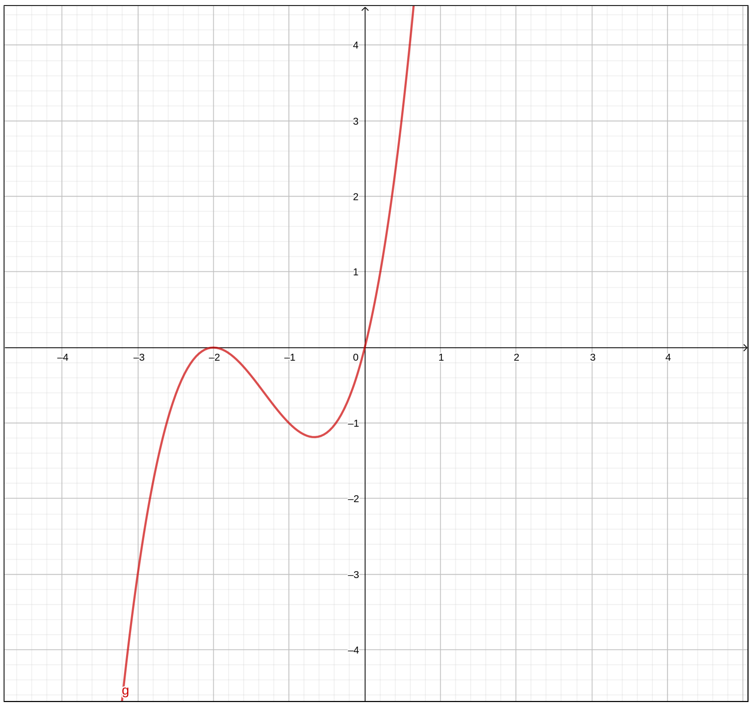
1. (*15 Points*) Given the graph of the following function $f(x)$, draw the graphs of the following.



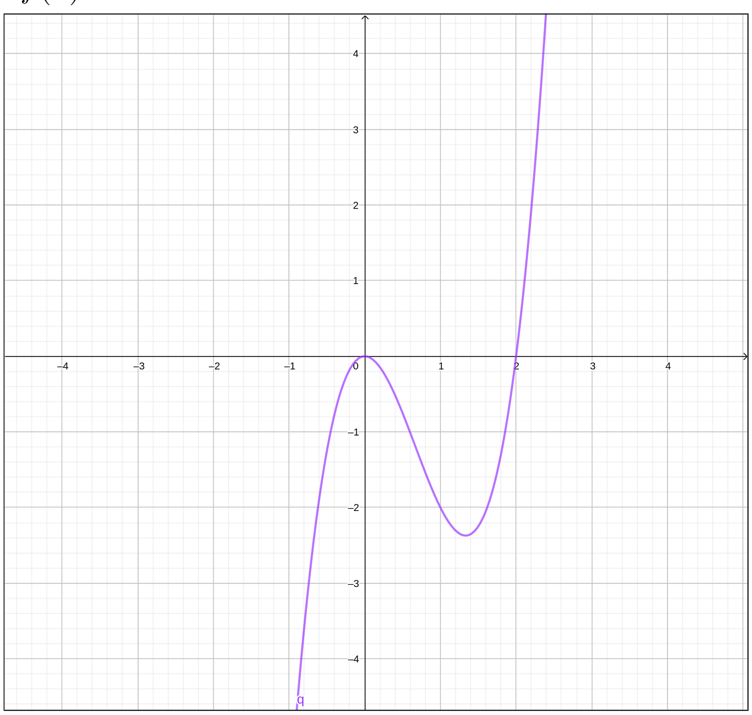
(a) $f(x) + 2$



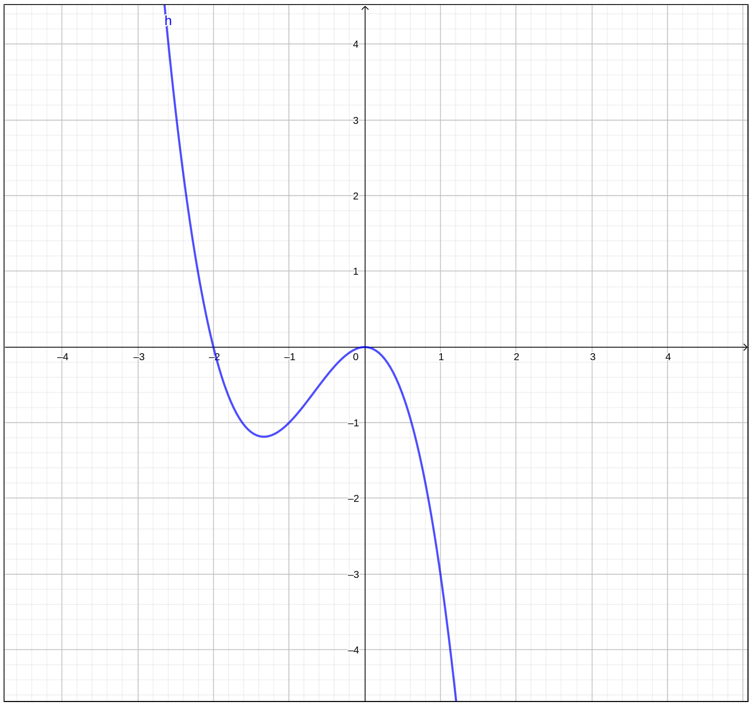
(b) $f(x+2)$



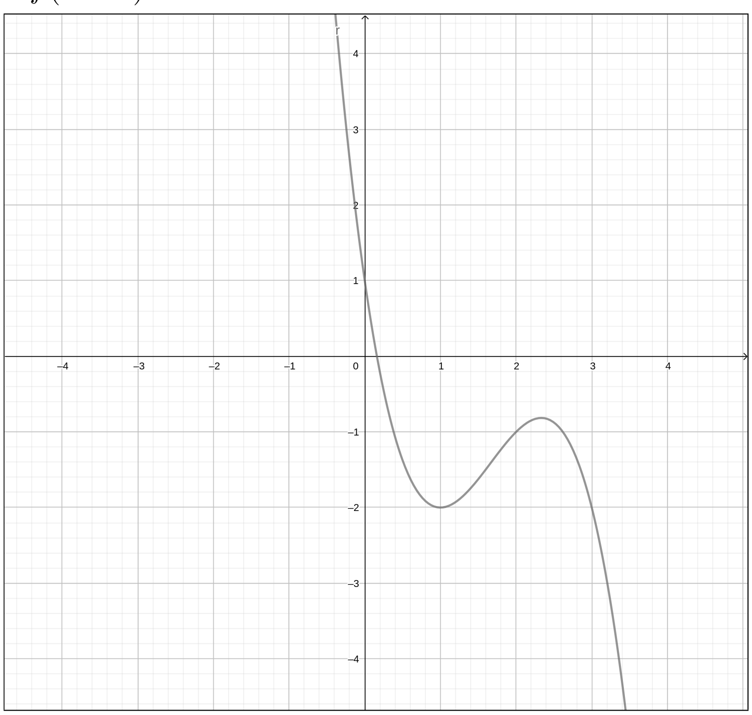
(c) $2f(x)$



(d) $f(-x)$



(e) $-f(x-1)-2$



2. (*10 Points*) Given the function $f(x) = x^2 + 4x + 19$.

   (a) Write the function in vertex form.
   Solution: $f(x) = x^2 + 4x + 19 = x^2 + 4x + 4 + 19 - 4 = (x + 2)^2 + 15$

   (b) What is the vertex of this parabola?
   Solution: $(-2, 15)$

   (c) Does this represent a maximum, minimum, or neither?
   Solution: Minimum

3. (*10 Points*) Show that $f(x) = x^3 - x^2 + x - 4$ has a zero in the interval $[1, 2]$.

   Solution: $f(1) = -3$ and $f(2) = 2$, so by the intermediate value theorem $f(x)$ has a zero in the interval $[1, 2]$.

4. (*10 Points*) Given the functions $f(x) = \sqrt{3x + 1}$ and $g(x) = \dfrac{x}{x + 2}$, find the following. You do not need to simplify your result.

   (a) $(f \circ g)(x)$
   Solution:

$$(f \circ g)(x) = f(g(x)) = \sqrt{3\left(\frac{x}{x+2}\right) + 1}$$

   (b) $(g \circ f)(x)$
   Solution:

$$(g \circ f)(x) = g(f(x)) = \frac{\sqrt{3x+1}}{\sqrt{3x+1} + 2}$$

5. (*15 Points*) Find the quotient and remainder when you divide $f(x) = x^3 + 2x^2 - 9x + 11$ by $p(x) = x^2 - x + 3$.

   Solution: $Q(x) = x + 3$ and $R(x) = -9x + 2$.

6. (*20 Points*) Given the function $f(x) = 3x^4 + x^3 - 36x^2 - 60x - 16$.

   (a) Using Descartes' Rule of Signs, determine the number of possible positive real zeros and the number of negative real zeros.
   Solution: By Descartes' Rule of Signs there exactly one positive real root and either 3 or 1 negative real roots.

   (b) Using Theorem on Rational Zeros of a Polynomial (a.k.a. The Rational Root Theorem), what are all the possible rational zeros of the function.
   Solution: $\pm 1, \pm 2, \pm 4, \pm 8, \pm 16, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{4}{3}, \pm\frac{8}{3}, \pm\frac{16}{3}$.

   (c) Given that -2 is a zero of multiplicity 2, factor $f(x)$ completely. What are the other two zeros?
   Solution: Synthetically divide -2 twice to get $f(x) = (x + 2)^2(3x^2 - 11x - 4) = (x + 2)^2(3x + 1)(x - 4)$. So the other zeros are 4 and $-\frac{1}{3}$.

(d) Find all the intervals where $f(x) > 0$, that is, the graph is above the x-axis. Find all the intervals where $f(x) < 0$, that is, the graph is below the x-axis.

Solution: $f(x) > 0$ on $(-\infty, -2) \cup (-2, -\frac{1}{3}) \cup (4, \infty)$ and $f(x) < 0$ on $(-\frac{1}{3}, 4)$.

7. (*20 Points*) Given the function $f(x) = \dfrac{x^2 + 3x + 2}{2x^2 - 4x}$

(a) Where are all the vertical asymptotes, if any?

Solution: $f(x) = \dfrac{x^2 + 3x + 2}{2x^2 - 4x} = \dfrac{(x+2)(x+1)}{2x(x-2)}$, so there are vertical asymptotes at $x = 0, 2$.

(b) Does the function have a horizontal asymptote? If so where?

Solution: There is a horizontal asymptote at $y = \frac{1}{2}$.

(c) Find all the x-intercepts for the function.

Solution: The x-intercepts are $x = -2, -1$.

(d) Find the y-intercept for the function.

Solution: None.

(e) For each vertical asymptote, determine if the function is going to ∞ or $-\infty$ on either side of the asymptote. Represent your answer as we did in class, for example, "as $x \to a^+$ then $f(x) \to \pm\infty$".

Solution:

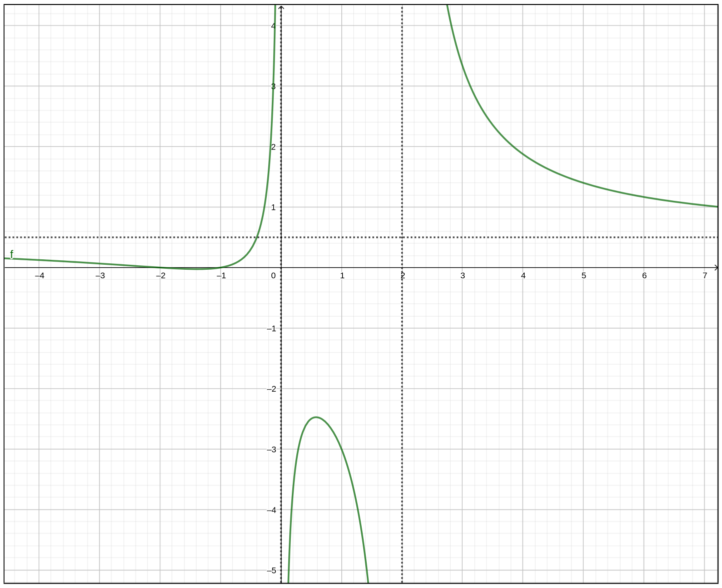
- As $x \to 0^-$ then $f(x) \to \infty$.
- As $x \to 0^+$ then $f(x) \to -\infty$.
- As $x \to 2^-$ then $f(x) \to -\infty$.
- As $x \to 2^+$ then $f(x) \to \infty$.

(f) Find all points where the graph crosses the horizontal asymptote.

Solution: Solve $\frac{x^2+3x+2}{2x^2-4x} = \frac{1}{2}$. So $x^2 + 3x + 2 = x^2 - 2x$, which gives $x = -\frac{2}{5} = -0.4$.

(g) Use all this information to sketch a graph of $f(x) = \dfrac{x^2 + 3x + 2}{2x^2 - 4x}$.

Solution: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