1. (*15 Points*) Find the exact value of the following.

   (a) $e^{2\ln(5)}$

   Solution: $e^{2\ln(5)} = e^{\ln(5^2)} = 5^2 = 25$.

   (b) $\log_{27}(3)$

   Solution: $y = \log_{27}(3)$ so $27^y = 3$ and hence $y = 1/3$.

   (c) $\log(1,000,000)$

   Solution: $\log(1,000,000) = \log(10^6) = 6$.

2. (*15 Points*) Solve the equation.

$$e^x + 2 = 8e^{-x}$$

   Solution: Let $t = e^x$,

$$\begin{aligned} e^x + 2 &= 8e^{-x} \\ (e^x)^2 + 2e^x &= 8 \\ (e^x)^2 + 2e^x - 8 &= 0 \\ t^2 + 2t - 8 &= 0 \\ (t+4)(t-2) &= 0 \end{aligned}$$

   So $e^x = -4$ or $e^x = 2$, the first has no real solutions and the second gives $x = \ln(2)$.

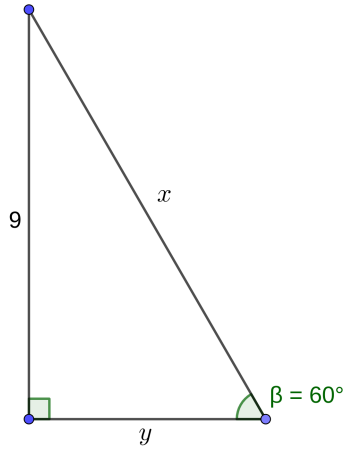
3. (*15 Points*) Solve the equation.

$$2\log(x+3) - \log(x+1) = 3\log(2)$$

   Solution:

$$\begin{aligned} 2\log(x+3) - \log(x+1) &= 3\log(2) \\ \log\left(\frac{(x+3)^2}{x+1}\right) &= \log(8) \\ \frac{(x+3)^2}{x+1} &= 8 \\ (x+3)^2 &= 8(x+1) \\ x^2 + 6x + 9 &= 8x + 8 \\ x^2 - 2x + 1 &= 0 \\ (x-1)^2 &= 0 \\ x &= 1 \end{aligned}$$

4. (*15 Points*) Find the exact values of x and y.



Solution: $\sin(\beta) = \frac{9}{x} = \frac{\sqrt{3}}{2}$, so $x = \frac{18}{\sqrt{3}} = 6\sqrt{3}$. $\cos(\beta) = \frac{y}{x} = \frac{1}{2}$, so $y = 3\sqrt{3}$.

5. (*15 Points*) Verify the identity by transforming the left-hand side into the right-hand side.

$$\frac{\cot(\theta) - 1}{1 - \tan(\theta)} = \cot(\theta)$$

Solution:

$$\begin{aligned}
\frac{\cot(\theta) - 1}{1 - \tan(\theta)} &= \frac{\frac{\cos(\theta)}{\sin(\theta)} - 1}{1 - \frac{\sin(\theta)}{\cos(\theta)}} \\
&= \frac{\frac{\cos(\theta) - \sin(\theta)}{\sin(\theta)}}{\frac{\cos(\theta) - \sin(\theta)}{\cos(\theta)}} \\
&= \frac{\cos(\theta) - \sin(\theta)}{\sin(\theta)} \cdot \frac{\cos(\theta)}{\cos(\theta) - \sin(\theta)} \\
&= \frac{\cos(\theta)}{\sin(\theta)} \\
&= \cot(\theta)
\end{aligned}$$

6. (*15 Points*) Find the exact values of the remaining trigonometric functions if $\csc(\theta) = \sqrt{13}/2$ and $\cot(\theta) = -3/2$.

Solution: Given $\csc(\theta) = \sqrt{13}/2$ and $\cot(\theta) = -3/2$,

$\sin(\theta) = 2/\sqrt{13}$

$\cos(\theta) = \cot(\theta)\sin(\theta) = -3/2 \cdot 2/\sqrt{13} = -3/\sqrt{13}$

$\tan(\theta) = -2/3$

$\sec(\theta) = -\sqrt{13}/3$

7. (*15 Points*) Find the amplitude, period, and phase shift and sketch the graph of

$$y = 2\cos\left(\frac{\pi}{2}x - 2\right)$$

Solution: Amplitude is 2, period is 4, and the phase shift is $\frac{4}{\pi}$.