1. (*15 Points*) Find the exact value of the following.

   (a) $e^{\ln(5)+\ln(2)}$
   Solution: $e^{\ln(5)+\ln(2)} = e^{\ln(10)} = 10$.

   (b) $\log_3(9)$
   Solution: $y = \log_3(9)$ so $3^y = 9$ and hence $y = 2$.

   (c) $\log(1,000,000,000)$
   Solution: $\log(1,000,000,000) = \log(10^9) = 9$.

2. (*15 Points*) Solve the equation.

$$e^x + 1 = 12e^{-x}$$

Solution: Let $t = e^x$,

$$\begin{aligned} e^x + 1 &= 12e^{-x} \\ (e^x)^2 + e^x &= 12 \\ (e^x)^2 + e^x - 12 &= 0 \\ t^2 + t - 12 &= 0 \\ (t+4)(t-3) &= 0 \end{aligned}$$

So $e^x = -4$ or $e^x = 3$, the first has no real solutions and the second gives $x = \ln(3)$.

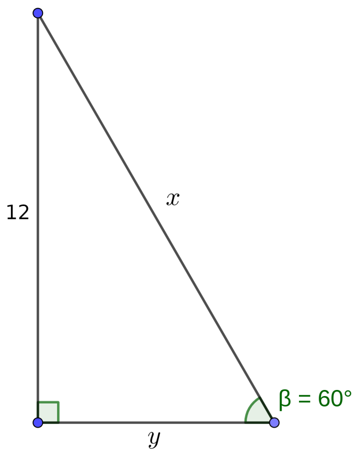
3. (*15 Points*) Solve the equation.

$$\log_4(x+1) = 2 + \log_4(3x-2)$$

Solution:

$$\begin{aligned} \log_4(x+1) &= 2 + \log_4(3x-2) \\ \log_4(x+1) - \log_4(3x-2) &= 2 \\ \log_4\left(\frac{x+1}{3x-2}\right) &= 2 \\ \frac{x+1}{3x-2} &= 16 \\ x+1 &= 16(3x-2) \\ x+1 &= 48x - 32 \\ 33 &= 47x \\ x &= \frac{33}{47} \end{aligned}$$

4. (*15 Points*) Find the exact values of x and y.



Solution: $\sin(\beta) = \frac{12}{x} = \frac{\sqrt{3}}{2}$, so $x = \frac{24}{\sqrt{3}} = 8\sqrt{3}$. $\cos(\beta) = \frac{y}{x} = \frac{1}{2}$, so $y = 4\sqrt{3}$.

5. (*15 Points*) Verify the identity by transforming the left-hand side into the right-hand side.

$$\frac{\tan(-\theta) + \cot(-\theta)}{\tan(\theta)} = -\csc^2(\theta)$$

Solution:

$$\begin{aligned} \frac{\tan(-\theta) + \cot(-\theta)}{\tan(\theta)} &= \frac{-\tan(\theta) - \cot(\theta)}{\tan(\theta)} \\ &= -\left(\frac{\tan(\theta) + \cot(\theta)}{\tan(\theta)}\right) \\ &= -\left(1 + \cot^2(\theta)\right) \\ &= -\csc^2(\theta) \end{aligned}$$

6. (*15 Points*) Find the exact values of the remaining trigonometric functions if $\sec(\theta) = \sqrt{13}/2$ and $\tan(\theta) = -3/2$.

Solution: Given $\sec(\theta) = \sqrt{13}/2$ and $\tan(\theta) = -3/2$,

$\cos(\theta) = 2/\sqrt{13}$

$\sin(\theta) = \tan(\theta)\cos(\theta) = -3/2 \cdot 2/\sqrt{13} = -3/\sqrt{13}$

$\cot(\theta) = -2/3$

$\csc(\theta) = -\sqrt{13}/3$

7. (*15 Points*) Find the amplitude, period, and phase shift and sketch the graph of

$$y = 3\sin\left(2x - \frac{\pi}{4}\right)$$

Solution: Amplitude is 3, period is π, and the phase shift is $\frac{\pi}{8}$.