1. (*10 Points*): Solve the following by completing the square.

$$4x^2 - 12x - 11 = 0$$

Solution: $x = 3/2 \pm \sqrt{5}$.

2. (*10 Points*): Solve the inequality and express the solution in terms of intervals.

$$-\frac{|6-5x|}{3} + 2 \geq 1$$

Solution: $\left[\frac{3}{5}, \frac{9}{5}\right]$

3. (*10 Points*): Find the point-slope, slope-intercept, and general forms of the equation of the line that passes through the points $(4, -5)$ and $(-3, 6)$.

Solution: $y + 5 = -\frac{11}{7}(x-4)$ or $y - 6 = -\frac{11}{7}(x+3)$, $y = -\frac{11}{7}x + \frac{9}{7}$, $7y + 11x = 9$.

4. (*10 Points*): Use the quadratic formula to find the zeros of f and find the maximum or minimum value of $f(x)$ where

$$f(x) = -3x^2 - 6x - 6$$

Solution: $x = -1 \pm i$, max at $x = -1$, $f(1) = -3$.

5. (*15 Points*): Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, express your answers in interval notation.

$$f(x) = -x^4 + 12x^2 - 27$$

Solution: $f(x) > 0$ on $(-3, -\sqrt{3}) \cup (\sqrt{3}, 3)$ and $f(x) < 0$ on $(-\infty, -3) \cup (-\sqrt{3}, \sqrt{3}) \cup (3, \infty)$.

6. (*15 Points*): Find all solutions of the equation.

$$x^3 + x^2 - 14x - 24 = 0$$

Solution: $x = -3, -2, 4$.

7. (*10 Points*): Find the quotient and remainder if $3x^3 + 2x - 4$ is divided by $x^2 + 1$.

Solution: $Q(x) = 3x$ and $R(x) = -x - 4$.

8. (*15 Points*): Given the function,

$$f(x) = \frac{2x^2 - 2x - 4}{x^2 - x - 12}$$

Find all asymptotes, intercepts, and holes. Also determine where the graph crosses any horizontal asymptotes.

Solution: y-intercept at $y = 1/3$, x-intercepts at $x = -1, 2$, vertical asymptotes at $x = -3, 4$, horizontal asymptote at $y = 2$, no holes, does not cross horizontal asymptote.

9. (*10 Points*): Solve the equation,

$$4^x \cdot \left(\frac{1}{2}\right)^{3-2x} = 8 \cdot (2^x)^2$$

Solution: $x = 3$.

10. (*15 Points*): Solve the equation,

$$\log_3(x-2) + \log_3(x-4) = 2$$

Solution: Solving gives $x = 3 \pm \sqrt{10}$ but the only solution is $x = 3 + \sqrt{10}$.

11. (*10 Points*): Write the expression as one logarithm.

$$5\log_7(x) - \frac{1}{2}\log_7(3x-4) - 3\log_7(5x+1)$$

Solution:

$$\log_7\left(\frac{x^5}{(5x+1)^3 \cdot \sqrt{3x+4}}\right)$$

12. (*10 Points*): Verify the following identity,

$$\cot(\theta) + \tan(\theta) = \csc(\theta)\sec(\theta)$$

Solution:

$$\cot(\theta) + \tan(\theta) = \frac{\cos(\theta)}{\sin(\theta)} + \frac{\sin(\theta)}{\cos(\theta)} = \frac{\sin^2(\theta) + \cos^2(\theta)}{\sin(\theta)\cos(\theta)} = \frac{1}{\sin(\theta)\cos(\theta)} = \csc(\theta)\sec(\theta)$$

13. (*15 Points*): Verify the following identity,

$$\frac{1}{\cot(x) - \cot(y)} = \frac{\sin(x)\sin(y)}{\sin(y-x)}$$

Solution:

$$\frac{1}{\cot(x) - \cot(y)} = \frac{1}{\frac{\cos(x)}{\sin(x)} - \frac{\cos(y)}{\sin(y)}} = \frac{1}{\frac{\cos(x)\sin(y) - \cos(y)\sin(x)}{\sin(x)\sin(y)}} = \frac{\sin(x)\sin(y)}{\cos(x)\sin(y) - \cos(y)\sin(x)} = RHS$$

14. (*10 Points*): Find the exact values of the following,

Solution:

(a) $\tan(5\pi/6) = -1/\sqrt{3}$

(b) $\cos(-11\pi/6) = \sqrt{3}/2$

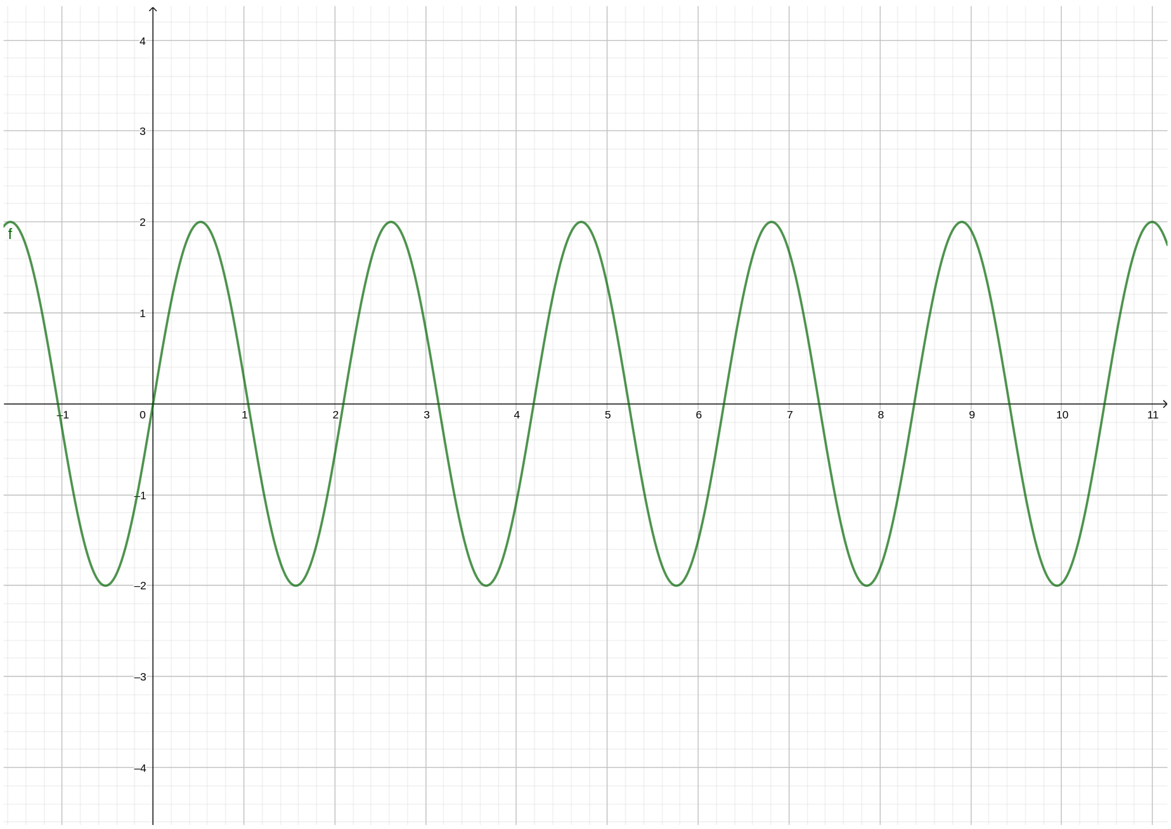
(c) $\sec(2\pi/3) = -2$

(d) $\cot(3\pi/4) = -1$

15. (*10 Points*): Find the amplitude, the period, and the phase shift and sketch the graph of,

$$f(x) = -2\sin(3x - \pi)$$

Solution: Amplitude is 2, period is $2\pi/3$, and phase shift is $\pi/3$.



16. (*10 Points*): Find all solutions of the equation,

$$4\sin^2(x)\cos(x) - 3\cos(x) - 4\sin^2(x) + 3 = 0$$

Solution:

$$4\sin^2(x)\cos(x) - 3\cos(x) - 4\sin^2(x) + 3 = (\cos(x) - 1)\left(4\sin^2(x) - 3\right)$$

$\cos(x) = 1$, so $x = 2\pi n$.

$\sin(x) = \pm\sqrt{3}/2$, so $x = \pi/3 + 2\pi n$ and $x = 2\pi/3 + 2\pi n$.

17. (*15 Points*): Find the exact values of,

(a) $\sin\left(\frac{11\pi}{12}\right)$ (b) $\cos\left(\frac{11\pi}{12}\right)$ (c) $\tan\left(\frac{11\pi}{12}\right)$

Note that $\frac{11\pi}{12} = \frac{2\pi}{3} + \frac{\pi}{4}$.

Solution:

$$\sin\left(\frac{11\pi}{12}\right) = \sin\left(\frac{2\pi}{3} + \frac{\pi}{4}\right) = \sin\left(\frac{2\pi}{3}\right)\cos\left(\frac{\pi}{4}\right) + \sin\left(\frac{\pi}{4}\right)\cos\left(\frac{2\pi}{3}\right)$$

So the value is

$$\frac{\sqrt{3}}{2}\frac{\sqrt{2}}{2} + \frac{-1}{2}\frac{\sqrt{2}}{2} = \frac{\sqrt{2}(\sqrt{3}-1)}{4} = \frac{\sqrt{6}-\sqrt{2}}{4}$$

$$\cos\left(\frac{11\pi}{12}\right) = \cos\left(\frac{2\pi}{3} + \frac{\pi}{4}\right) = \cos\left(\frac{2\pi}{3}\right)\cos\left(\frac{\pi}{4}\right) - \sin\left(\frac{\pi}{4}\right)\sin\left(\frac{2\pi}{3}\right)$$

So the value is

$$\frac{-1}{2}\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\frac{\sqrt{3}}{2} = \frac{\sqrt{2}(-1-\sqrt{3})}{4} = \frac{-\sqrt{6}-\sqrt{2}}{4}$$

$$\tan\left(\frac{11\pi}{12}\right) = \frac{\sin\left(\frac{11\pi}{12}\right)}{\cos\left(\frac{11\pi}{12}\right)} = \frac{\sqrt{6}-\sqrt{2}}{-\sqrt{6}-\sqrt{2}} = -\frac{\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}$$