1. (*10 Points*): Solve the following by completing the square.

$$3x^2 - 8x - 7 = 0$$

Solution: $x = \dfrac{4 \pm \sqrt{37}}{3}$.

2. (*10 Points*): Solve the inequality and express the solution in terms of intervals.

$$\frac{|3x-4|}{3} + 5 > 10$$

Solution: $\left(-\infty, -\dfrac{11}{3}\right) \cup \left(\dfrac{19}{3}, \infty\right)$

3. (*10 Points*): Find the point-slope, slope-intercept, and general forms of the equation of the line that passes through the points $(-1, 7)$ and $(3, 2)$.

Solution: $y - 7 = -\frac{5}{4}(x+1)$ or $y - 2 = -\frac{5}{4}(x-3)$, $y = -\frac{5}{4}x + \frac{23}{4}$, $4y + 5x = 23$.

4. (*10 Points*): Use the quadratic formula to find the zeros of f and find the maximum or minimum value of $f(x)$ where

$$f(x) = 2x^2 - 3x + 3$$

Solution: $x = \frac{3 \pm i\sqrt{15}}{4}$, min at $x = 3/4$, $f(3/4) = 15/8$.

5. (*15 Points*): Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, express your answers in interval notation.

$$f(x) = x^4 - 11x^2 + 28$$

Solution: $f(x) > 0$ on $(-\infty, -\sqrt{7}) \cup (-2, 2) \cup (\sqrt{7}, \infty)$ and $f(x) < 0$ on $(-\sqrt{7}, -2) \cup (2, \sqrt{7})$.

6. (*15 Points*): Find all solutions of the equation.

$$x^3 - 6x^2 + 3x + 10 = 0$$

Solution: $x = -1, 2, 5$.

7. (*10 Points*): Find the quotient and remainder if $2x^3 + x - 5$ is divided by $x^2 + x + 1$.

Solution: $Q(x) = 2x - 2$ and $R(x) = x + 3$.

8. (*15 Points*): Given the function,

$$f(x) = \frac{4x^2 - 4x - 3}{x^2 + x - 2}$$

Find all asymptotes, intercepts, and holes. Also determine where the graph crosses any horizontal asymptotes.

Solution: y-intercept at $y = 3/2$, x-intercepts at $x = -1/2, 3/2$, vertical asymptotes at $x = -2, 1$, horizontal asymptote at $y = 4$, no holes, crosses horizontal asymptote at $x = 5/8$.

9. (*10 Points*): Solve the equation,

$$8^{2x} \cdot \left(\frac{1}{4}\right)^{x-2} = 4^{-x} \cdot \left(\frac{1}{2}\right)^{2-x}$$

Solution: $x = -6/5$.

10. (*15 Points*): Solve the equation,

$$\log_2(x+1) + \log_2(x-3) = 3$$

Solution: Solving gives $x = 1 \pm 2\sqrt{3}$ but the only solution is $x = 1 + 2\sqrt{3}$.

11. (*10 Points*): Write the expression as one logarithm.

$$7\log_5(x) - 3\log_5(4x-5) + \frac{1}{3}\log_5(x+1)$$

Solution:

$$\log_5\left(\frac{x^7\sqrt[3]{x+1}}{(4x-5)^3}\right)$$

12. (*10 Points*): Verify the following identity,

$$\frac{\sec(x) - \cos(x)}{\tan(x)} = \sin(x)$$

Solution:

$$\frac{\sec(x) - \cos(x)}{\tan(x)} = \frac{1/\cos(x) - \cos(x)}{\sin(x)/\cos(x)} = \frac{1 - \cos^2(x)}{\sin(x)} = \frac{\sin^2(x)}{\sin(x)} = \sin(x)$$

13. (*15 Points*): Verify the following identity,

$$\tan\left(x + \frac{3\pi}{4}\right) = \frac{\tan(x) - 1}{1 + \tan(x)}$$

Solution:

$$\tan\left(x + \frac{3\pi}{4}\right) = \frac{\sin(x + 3\pi/4)}{\cos(x + 3\pi/4)} = \frac{\sin(x)\cos(3\pi/4) + \sin(3\pi/4)\cos(x)}{\cos(x)\cos(3\pi/4) - \sin(x)\sin(3\pi/4)}$$

$$= \frac{\sin(x)(-\sqrt{2}/2) + (\sqrt{2}/2)\cos(x)}{\cos(x)(-\sqrt{2}/2) - \sin(x)(\sqrt{2}/2)} = \frac{-\sin(x) + \cos(x)}{-\cos(x) - \sin(x)} = RHS$$

14. (*10 Points*): Find the exact values of the following,

Solution:

(a) $\tan(\pi/6) = 1/\sqrt{3}$

(b) $\cos(-5\pi/6) = -\sqrt{3}/2$

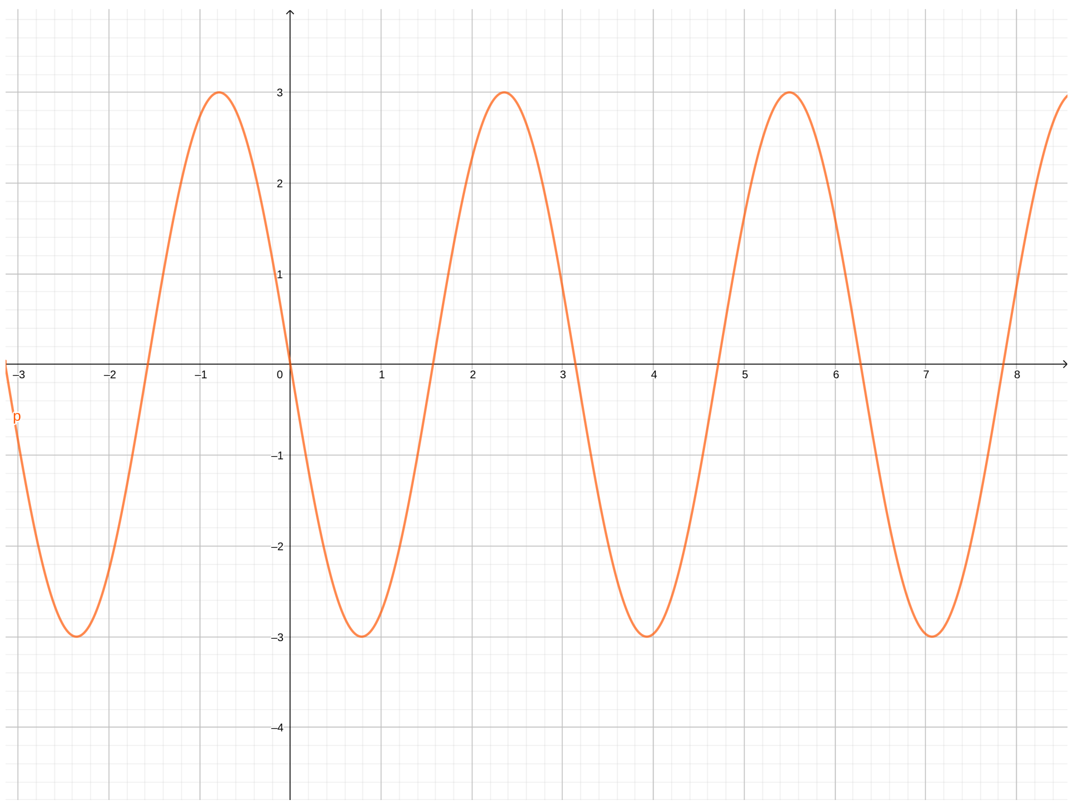
(c) $\sec(\pi/4) = \sqrt{2}$

(d) $\cot(\pi/6) = \sqrt{3}$

15. (*10 Points*): Find the amplitude, the period, and the phase shift and sketch the graph of,

$$f(x) = 3\cos\left(2x + \frac{\pi}{2}\right)$$

Solution: Amplitude is 3, period is π, and phase shift is $-\pi/4$.



16. (*10 Points*): Find all solutions of the equation,

$$2\cos^2(x)\sin(x) - 2\sin(x) + \cos^2(x) - 1 = 0$$

Solution:

$$2\cos^2(x)\sin(x) - 2\sin(x) + \cos^2(x) - 1 = (\cos^2(x) - 1)(2\sin(x) + 1)$$

$\cos(x) = \pm 1$, so $x = \pi n$.

$\sin(x) = -1/2$, so $x = 7\pi/6 + 2\pi n$ and $x = 11\pi/6 + 2\pi n$.

17. (*15 Points*): Find the exact values of,

(a) $\sin\left(\frac{\pi}{12}\right)$ (b) $\cos\left(\frac{\pi}{12}\right)$ (c) $\tan\left(\frac{\pi}{12}\right)$

Note that $\frac{\pi}{12} = \frac{\pi}{3} - \frac{\pi}{4}$.

Solution:

$$\sin\left(\frac{\pi}{12}\right) = \sin\left(\frac{\pi}{3} - \frac{\pi}{4}\right) = \sin\left(\frac{\pi}{3}\right)\cos\left(\frac{\pi}{4}\right) - \sin\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{3}\right)$$

So the value is

$$\frac{\sqrt{3}}{2}\frac{\sqrt{2}}{2} - \frac{1}{2}\frac{\sqrt{2}}{2} = \frac{\sqrt{2}(\sqrt{3}-1)}{4} = \frac{\sqrt{6}-\sqrt{2}}{4}$$

$$\cos\left(\frac{\pi}{12}\right) = \cos\left(\frac{\pi}{3} - \frac{\pi}{4}\right) = \cos\left(\frac{\pi}{3}\right)\cos\left(\frac{\pi}{4}\right) + \sin\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{3}\right)$$

So the value is

$$\frac{1}{2}\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\frac{\sqrt{3}}{2} = \frac{\sqrt{2}(1+\sqrt{3})}{4} = \frac{\sqrt{6}+\sqrt{2}}{4}$$

$$\tan\left(\frac{\pi}{12}\right) = \frac{\sin\left(\frac{\pi}{12}\right)}{\cos\left(\frac{\pi}{12}\right)} = \frac{\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}$$