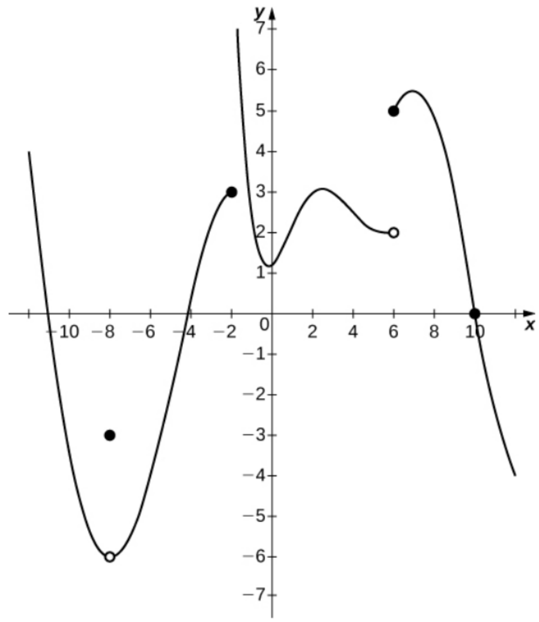
1. (*10 Points*) Given the graph of the the function f below, answer the following.



(a) $\lim_{x\to -8} f(x) = -6$

(b) $f(-8) = -3$

(c) $\lim_{x\to -2} f(x) =$ Does not exist.

(d) $\lim_{x\to -2^+} f(x) = \infty$

(e) $\lim_{x\to -2^-} f(x) = 3$

(f) $f(-2) = 3$

(g) $\lim_{x\to 6} f(x) =$ Does not exist.

(h) $\lim_{x\to 6^+} f(x) = 5$

(i) $\lim_{x\to 6^-} f(x) = 2$

(j) $f(6) = 5$

(k) List all points of discontinuity. For each, state the type of discontinuity and state if the function is continuous from the left or right at that point.

- $x = -8$: Removable, not continuous from the left or right.
- $x = -2$: Infinite, continuous from the left.
- $x = 6$: Jump, continuous from the right.

2. (*10 Points*) Sketch the graph of an example of a function f that satisfies all of the given conditions.

   (a) $\lim_{x \to -3} f(x) = 2$

   (b) $f(-3) = -2$

   (c) $\lim_{x \to 0^+} f(x) = 1$

   (d) $\lim_{x \to 0^-} f(x) = \infty$

   (e) $f(0)$ Does not exist.

   (f) $\lim_{x \to 1} f(x)$ Does not exist.

   (g) $\lim_{x \to 2^+} f(x) = 4$

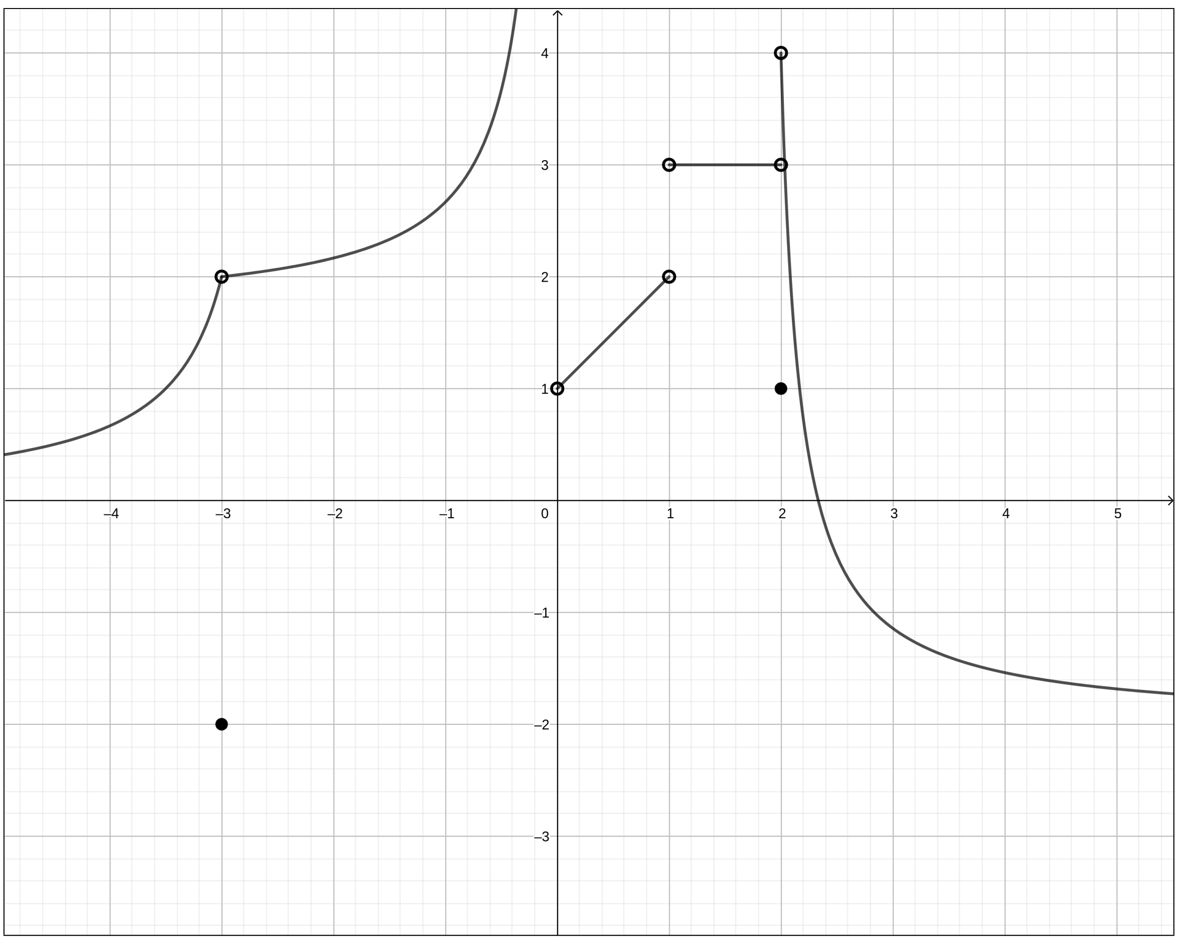
   (h) $\lim_{x \to 2^-} f(x) = 3$

   (i) $f(2) = 1$

   (j) $\lim_{x \to \infty} f(x) = -2$

   (k) $\lim_{x \to -\infty} f(x) = 0$

Solution:



3. (*10 Points*) Find the following limit using limit laws. Keep your answer in exact form.

$$\lim_{x\to -1} \frac{x+1}{x^3+1}$$

Solution:

$$\begin{aligned}
\lim_{x\to -1} \frac{x+1}{x^3+1} &= \lim_{x\to -1} \frac{x+1}{(x+1)(x^2-x+1)} \\
&= \lim_{x\to -1} \frac{1}{x^2-x+1} = \frac{1}{3}
\end{aligned}$$

4. (*10 Points*) Find the following limit using limit laws. Keep your answer in exact form.

$$\lim_{h\to 0} \frac{(-2+h)^{-1}+2^{-1}}{h}$$

Solution:

$$\begin{aligned}
\lim_{h\to 0} \frac{(-2+h)^{-1}+2^{-1}}{h} &= \lim_{h\to 0} \frac{\frac{1}{h-2}+\frac{1}{2}}{h} \\
&= \lim_{h\to 0} \frac{\frac{2+h-2}{2(h-2)}}{h} \\
&= \lim_{h\to 0} \frac{\frac{h}{2(h-2)}}{h} \\
&= \lim_{h\to 0} \frac{h}{2(h-2)h} \\
&= \lim_{h\to 0} \frac{1}{2(h-2)} = -\frac{1}{4}
\end{aligned}$$

5. (*10 Points*) Find the following limit using limit laws. Keep your answer in exact form.

$$\lim_{x\to 2} \frac{x^2-4x+4}{x^4-3x^2-4}$$

Solution:

$$\begin{aligned}
\lim_{x\to 2} \frac{x^2-4x+4}{x^4-3x^2-4} &= \lim_{x\to 2} \frac{(x-2)^2}{(x^2-4)(x^2+1)} \\
&= \lim_{x\to 2} \frac{(x-2)^2}{(x+2)(x-2)(x^2+1)} \\
&= \lim_{x\to 2} \frac{x-2}{(x+2)(x^2+1)} = 0
\end{aligned}$$

6. (*10 Points*) Find the following limit using limit laws. Keep your answer in exact form.

$$\lim_{t\to 0} \left(\frac{1}{t} - \frac{1}{t^2+t}\right)$$

Solution:

$$\begin{aligned}\lim_{t\to 0}\left(\frac{1}{t}-\frac{1}{t^2+t}\right) &= \lim_{t\to 0}\frac{t+1-1}{t(t+1)}\\ &= \lim_{t\to 0}\frac{t}{t(t+1)}\\ &= \lim_{t\to 0}\frac{1}{t+1} = 1\end{aligned}$$

7. (*10 Points*) Show that there is a solution of the equation

$$\sin(x)-\frac{1}{4}=\cos(x)+\frac{1}{4}$$

in the interval $\left(\frac{\pi}{4}, \frac{2\pi}{3}\right)$

Solution: A solution to

$$\sin(x)-\frac{1}{4}=\cos(x)+\frac{1}{4}$$

is equivalent to a solution to

$$\cos(x)-\sin(x)+\frac{1}{2}=0$$

Let $f(x)=\cos(x)-\sin(x)+\frac{1}{2}$, then $f(\pi/4)=\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}+\frac{1}{2}=\frac{1}{2}$ and $f(2\pi/3)=-\frac{1}{2}-\frac{\sqrt{3}}{2}+\frac{1}{2}=-\frac{\sqrt{3}}{2}$. Since $f(x)$ is continuous for all real numbers, the intermediate value theorem implies that there is a value $\frac{\pi}{4}<c<\frac{2\pi}{3}$ with $f(c)=0$.

8. (*10 Points*) Find the following limits. Keep your answers in exact form.

$$\lim_{x\to\infty}\frac{x+3}{\sqrt{2x^2-1}} \qquad \lim_{x\to-\infty}\frac{x+3}{\sqrt{2x^2-1}}$$

Solution:

$$\begin{aligned}\lim_{x\to\infty}\frac{x+3}{\sqrt{2x^2-1}} &= \lim_{x\to\infty}\frac{\frac{x+3}{x}}{\frac{\sqrt{2x^2-1}}{x}}\\ &= \lim_{x\to\infty}\frac{1+\frac{3}{x}}{\sqrt{\frac{2x^2-1}{x^2}}}\\ &= \lim_{x\to\infty}\frac{1+\frac{3}{x}}{\sqrt{2-\frac{1}{x^2}}}=\frac{1}{\sqrt{2}}\end{aligned}$$

$$\begin{aligned}\lim_{x\to-\infty}\frac{x+3}{\sqrt{2x^2-1}} &= \lim_{x\to-\infty}\frac{\frac{x+3}{x}}{\frac{\sqrt{2x^2-1}}{x}}\\ &= \lim_{x\to-\infty}-\frac{1+\frac{3}{x}}{\sqrt{\frac{2x^2-1}{x^2}}}\\ &= \lim_{x\to-\infty}-\frac{1+\frac{3}{x}}{\sqrt{2-\frac{1}{x^2}}}=-\frac{1}{\sqrt{2}}\end{aligned}$$

9. (*10 Points*) Find the following limits. Keep your answers in exact form.

$$\lim_{x\to\infty}\left(\sqrt{4x^2+3x}+2x\right) \qquad \lim_{x\to-\infty}\left(\sqrt{4x^2+3x}+2x\right)$$

Solution: The first limit is infinite since both terms are increasing without bound and we are adding them together.

$$\lim_{x\to\infty}\left(\sqrt{4x^2+3x}+2x\right)=\infty$$

The second limit is of the indeterminate form $\infty-\infty$.

$$\begin{aligned}
\lim_{x\to-\infty}\left(\sqrt{4x^2+3x}+2x\right) &= \lim_{x\to-\infty}\left(\sqrt{4x^2+3x}+2x\right)\cdot\frac{\sqrt{4x^2+3x}-2x}{\sqrt{4x^2+3x}-2x}\\
&= \lim_{x\to-\infty}\frac{(4x^2+3x)-4x^2}{\sqrt{4x^2+3x}-2x}\\
&= \lim_{x\to-\infty}\frac{3x}{\sqrt{4x^2+3x}-2x}\\
&= \lim_{x\to-\infty}\frac{3x}{\sqrt{x^2}\sqrt{4+\frac{3}{x}}-2x}\\
&= \lim_{x\to-\infty}\frac{3x}{|x|\sqrt{4+\frac{3}{x}}-2x}\\
&= \lim_{x\to-\infty}\frac{3x}{-x\sqrt{4+\frac{3}{x}}-2x}\\
&= \lim_{x\to-\infty}\frac{3x}{x\left(-\sqrt{4+\frac{3}{x}}-2\right)}\\
&= \lim_{x\to-\infty}\frac{3}{-\sqrt{4+\frac{3}{x}}-2}=-\frac{3}{4}
\end{aligned}$$

10. (*10 Points*) Prove that

$$\lim_{x\to0}\left(\sqrt{x^3+x^2}\right)\sin\left(\frac{\pi}{x}\right)=0$$

Solution: Since $-1\le\sin(\pi/x)\le1$ we have

$$-\sqrt{x^3+x^2}\le\left(\sqrt{x^3+x^2}\right)\sin\left(\frac{\pi}{x}\right)\le\sqrt{x^3+x^2}$$

So

$$0=\lim_{x\to0}-\sqrt{x^3+x^2}\le\lim_{x\to0}\left(\sqrt{x^3+x^2}\right)\sin\left(\frac{\pi}{x}\right)\le\lim_{x\to0}\sqrt{x^3+x^2}=0$$

Hence by the squeeze theorem,

$$\lim_{x\to0}\left(\sqrt{x^3+x^2}\right)\sin\left(\frac{\pi}{x}\right)=0$$