Name: ______________________________

- Write all of your responses on these exam pages. If you need more space for your answers please use the backs of the exam pages.
- Make sure that you show all of your work, answers without supporting work will receive no credit.
- **No calculation devices are to be used on this exam.**

1. (*15 Points*) Find the following limits using limit laws. You may not use l'Hospital's/Bernoulli's Rule. Keep your answers in exact form.

   (a) $\lim\limits_{x \to 1} \dfrac{x^2 - 1}{x^3 - 1}$

   (b) $\lim\limits_{x \to 5} \sqrt[3]{\dfrac{x^2 - x + 5}{3x - 2}}$

(c) $\lim_{x \to \infty} \sqrt{x^2 + 4x + 1} - x$

2. (*10 Points*) Prove that

$$\lim_{x \to 0} \left(\sqrt[5]{x^2 + x} \right) \cos\left(\frac{17}{x} \right) = 0$$

3. (*15 Points*) Using the definition of the derivative, find $f'(x)$ for

$$f(x) = \frac{x^2}{x+2}$$

4. (*20 Points*) Using the derivative rules, find $f'(x)$ for each of the following functions. You do not need to simplify your results.

   (a) $f(x) = e^x \sin(x) \tan(x)$

   (b) $f(x) = \dfrac{2x^2 + 3}{x^3 - x + 1}$

(c) $f(x) = \sqrt[7]{4x - 2 + \cos x}$

(d) $f(x) = \tan^{-1}\left(\dfrac{x \sec(x)}{e^{\cos(x)}}\right)$

5. (*15 Points*) Find a parabola with equation $y = ax^2 + bx + c$ that has slope 4 at $x = 1$, slope -8 at $x = -1$, and passes through the point $(2, 15)$.

6. (*15 Points*) Find the equation of the tangent line, in slope-intercept form, to

$$xy^2 = (x+1)^2(5-y^2)$$

at the point $(-4, -3)$. Keep your answer in exact form.

7. (*15 Points*) At noon, ship A is 150 km west of ship B. Ship A is sailing east at 35 km/h and ship B is sailing north at 25 km/h. How fast is the distance between the ships changing at 4:00 PM? Keep your answer in exact form but simplify the final number as far as possible, that is, down to fractions, trigonometric, exponential, logarithmic, and roots of single numbers.

8. (*25 Points*) For the following function

$$f(x) = \frac{x^2 - 1}{x^2 - 4}$$

(a) Find the domain of the function.

(b) Find the x and y intercepts of the function.

(c) Test for even/odd symmetry.

(d) Find the vertical, horizontal, and tilted asymptotes.

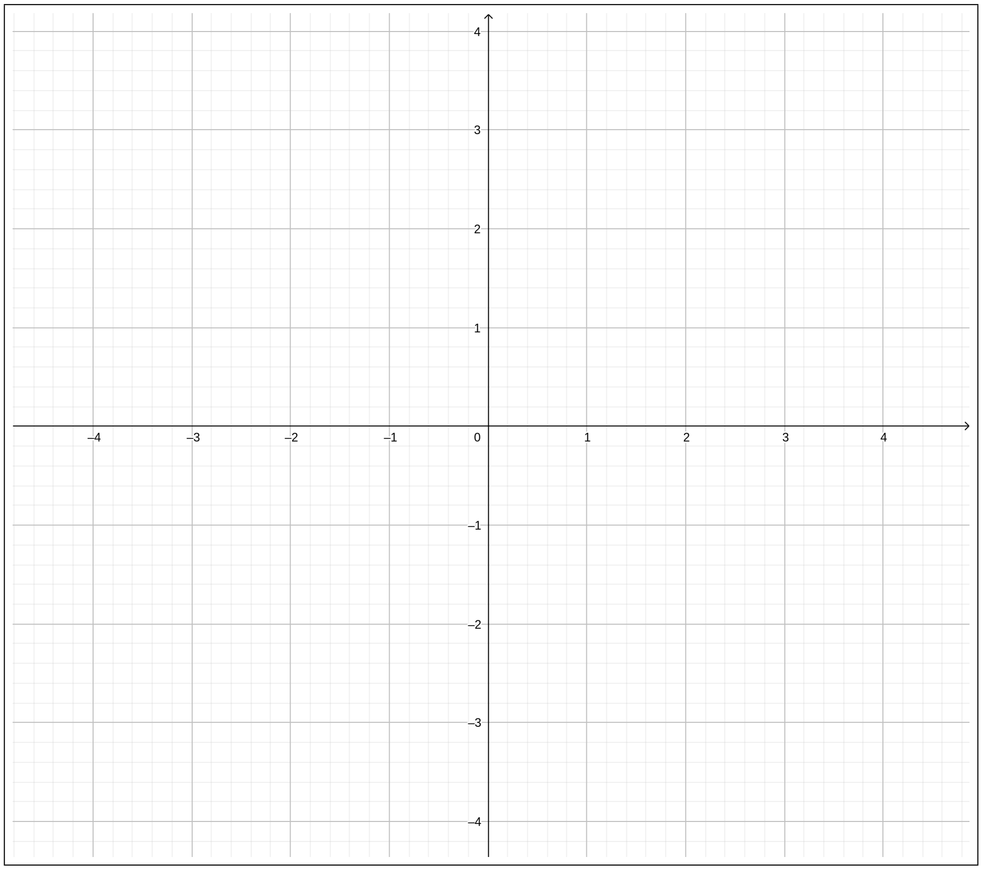
(e) Find the intervals of increase or decrease.

(f) Find the local maximum and minimum values.

(g) Find the intervals of concave up or concave down.

(h) Find the inflection points.

(i) Make a sketch of the graph given the information about the properties of the function.



9. (*10 Points*) Find the absolute maximum and minimum of

$$f(x) = x^3 - 6x^2 + 9x + 1$$

on the interval $[-2, 2]$. Keep your answers in exact form.

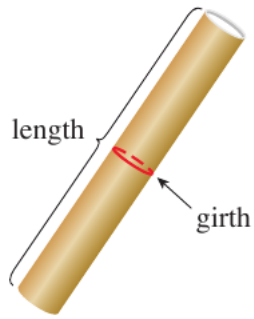
10. (*15 Points*) Find the following limits,

(a) $\displaystyle\lim_{x\to\infty} \frac{\ln(\sqrt{x})}{x^2}$

(b) $\displaystyle\lim_{x\to 0} \frac{x^2\sin(x)}{\sin(x)-x}$

(c) $\lim_{x \to 0} (\cos(x))^{1/x^2}$

11. (*15 Points*): A package to be mailed using the US postal service may not measure more than 108 inches in length plus girth. (Length is the longest dimension and girth is the largest distance around the package, perpendicular to the length.) Find the dimensions (radius and length) of the cylindrical mailing tube of greatest volume that may be mailed using the US postal service.



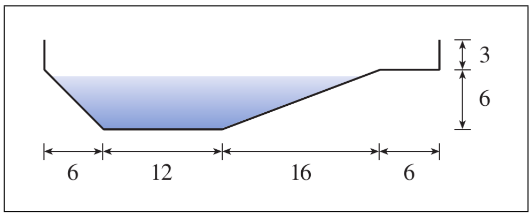
12. (*10 Points*) Set up the Newton's Method formula for the n^{th} approximation to a root of the function $f(x) = x^2 + \sin(\pi x^3)$.

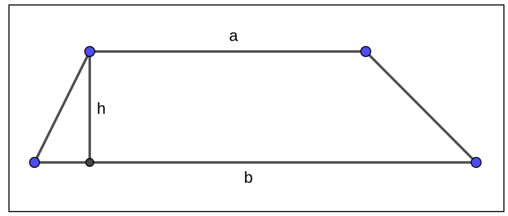
13. (*10 Points*) Find the function $f(x)$ given that $f''(x) = 2 + 12x - 12x^2$, $f(0) = 4$ and $f'(0) = 12$.

14. (*10 Points*) Find the function $f(x)$ given that $f''(x) = e^x - 2\sin(x)$, $f(0) = 3$ and $f(\pi/2) = 0$.

15. **Extra Credit:** (*10 Points*) Do one and only one of the following.

(a) A swimming pool is 20 ft wide, 40 ft long, 3 ft deep at the shallow end, and 9 ft deep at its deepest point. A cross-section is shown in the left figure. If the pool is being filled at a rate of 0.8 ft^3/min, how fast is the water level rising when the depth at the deepest point is 5 ft? The area of a trapezoid is the average of the two bases (parallel sides) times the height. So in the right figure below we have $A = h \cdot \frac{a+b}{2}$.





(b) The upper right-hand corner of a piece of paper, 12 inches by 8 inches, as in the figure, is folded over to the bottom edge. How would you fold the paper so as to minimize the length of the fold? That is, how would you choose x to minimize y?