1. Find the following limit using limit laws. Keep your answer in exact form.

$$\lim_{x\to -2} \frac{x^2 - x - 6}{3x^2 + 5x - 2}$$

**Solution:**

$$\lim_{x\to -2} \frac{x^2 - x - 6}{3x^2 + 5x - 2} = \lim_{x\to -2} \frac{(x+2)(x-3)}{(x+2)(3x-1)} = \lim_{x\to -2} \frac{x-3}{3x-1} = \frac{-5}{-7} = \frac{5}{7}$$

2. Find the following limit using limit laws. Keep your answer in exact form.

$$\lim_{x\to \infty} \frac{x - x^3}{2 - x^2 + 3x^3}$$

**Solution:**

$$\lim_{x\to \infty} \frac{x - x^3}{2 - x^2 + 3x^3} \cdot \frac{\frac{1}{x^3}}{\frac{1}{x^3}} = \lim_{x\to \infty} \frac{\frac{1}{x^2} - 1}{\frac{2}{x^3} - \frac{1}{x} + 3} = -\frac{1}{3}$$

3. Explain why the function

$$f(x) = \frac{\cos(x^2)}{1 - e^x}$$

is continuous at every number in its domain and state the domain.

**Solution:** The function is continuous at every number in its domain since it is an algebraic combination of trigonometric, polynomial, and exponential functions. The domain of this function is any place where $1 - e^x \neq 0$, hence for all real numbers $x$ except for $x = 0$.

4. State the following definitions,

   (a) State the definition of the derivative of a function $f(x)$ at the point $x = a$, that is, $f'(a)$.
   **Solution:**

$$f'(a) = \lim_{x\to a} \frac{f(x) - f(a)}{x - a} \qquad \text{or} \qquad f'(a) = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h}$$

   (b) State the definition of the derivative of a function $f(x)$, that is, $f'(x)$.
   **Solution:**

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

5. Use the definition of the derivative to find the derivative of the function,

$$f(x) = \frac{1}{x^2 - 4}$$

**Solution:**

$$\begin{aligned}
f'(x) &= \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} \\
&= \lim_{h\to 0} \frac{\frac{1}{(x+h)^2-4} - \frac{1}{x^2-4}}{h} \\
&= \lim_{h\to 0} \frac{(x^2-4) - ((x+h)^2-4)}{h(x^2-4)((x+h)^2-4)} \\
&= \lim_{h\to 0} \frac{(x^2-4) - (x^2+2xh+h^2-4)}{h(x^2-4)((x+h)^2-4)} \\
&= \lim_{h\to 0} \frac{x^2-4-x^2-2xh-h^2+4}{h(x^2-4)((x+h)^2-4)} \\
&= \lim_{h\to 0} \frac{-2xh-h^2}{h(x^2-4)((x+h)^2-4)} \\
&= \lim_{h\to 0} \frac{h(-2x-h)}{h(x^2-4)((x+h)^2-4)} \\
&= \lim_{h\to 0} \frac{-2x-h}{(x^2-4)((x+h)^2-4)} \\
&= \frac{-2x}{(x^2-4)^2}
\end{aligned}$$