1. (*45 Points*): Find the first derivatives of the following functions. You do not need to simplify your answers.

   (a) $f(x) = x^{3/2} + x^{-3}$

   Solution: $f'(x) = \dfrac{3}{2}x^{1/2} - 3x^{-4}$

   (b) $f(x) = (3x^2 - 5x)e^x$

   Solution: $f'(x) = (3x^2 - 5x)e^x + (6x - 5)e^x$

   (c) $f(x) = \dfrac{x^2 e^x}{x^2 + e^x}$

   Solution: $f'(x) = \dfrac{(x^2 + e^x)(x^2 e^x + 2xe^x) - (x^2 e^x)(2x + e^x)}{(x^2 + e^x)^2}$

   (d) $f(x) = \dfrac{x}{2 - \tan(x)}$

   Solution: $f'(x) = \dfrac{2 - \tan(x) - x(-\sec^2(x))}{(2 - \tan(x))^2} = \dfrac{2 - \tan(x) + x\sec^2(x)}{(2 - \tan(x))^2}$

   (e) $f(x) = \sin\left(\dfrac{e^x}{1 + e^x}\right)$

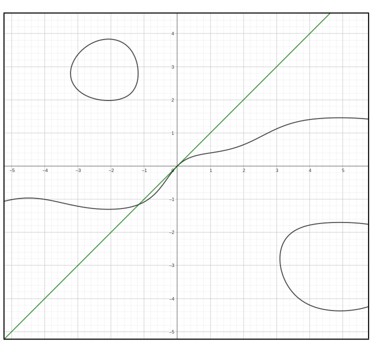
   Solution: $f'(x) = \cos\left(\dfrac{e^x}{1 + e^x}\right)\dfrac{(1 + e^x)e^x - e^{2x}}{(1 + e^x)^2} = \cos\left(\dfrac{e^x}{1 + e^x}\right)\dfrac{e^x}{(1 + e^x)^2}$

2. (*15 Points*): Find the derivative of the following implicit relationship.

$$ye^{\sin(x)} = x\cos(y)$$

   Then find the tangent line to the curve at the point $(0, 0)$.

   Solution:



$$\begin{aligned} ye^{\sin(x)} &= x\cos(y) \\ ye^{\sin(x)}\cos(x) + y'e^{\sin(x)} &= -x\sin(y)y' + \cos(y) \\ y'e^{\sin(x)} + x\sin(y)y' &= \cos(y) - ye^{\sin(x)}\cos(x) \\ y'(e^{\sin(x)} + x\sin(y)) &= \cos(y) - ye^{\sin(x)}\cos(x) \\ y' &= \frac{\cos(y) - ye^{\sin(x)}\cos(x)}{e^{\sin(x)} + x\sin(y)} \end{aligned}$$

At $(0, 0)$, $y' = \frac{\cos(0) - 0 \cdot e^{\sin(0)}\cos(0)}{e^{\sin(0)} + 0 \cdot \sin(0)} = 1$, so the equation of the tangent line is $y = x$.

3. (*10 Points*): Find the derivative of $f(x) = x^x$ using logarithmic differentiation.

   Solution:

$$\begin{aligned} y &= x^x \\ \ln(y) &= \ln(x^x) = x\ln(x) \\ \frac{y'}{y} &= 1 + \ln(x) \\ y' &= y(1+\ln(x)) = x^x(1+\ln(x)) \end{aligned}$$

4. (*10 Points*): Find the following limit.

$$\lim_{x\to 0} \frac{\tan(2x)}{x}$$

   Solution:

$$\lim_{x\to 0} \frac{\tan(2x)}{x} = \lim_{x\to 0} \frac{\sin(2x)}{x\cos(2x)} = \lim_{x\to 0} \frac{2\sin(2x)}{2x\cos(2x)} = \left(2\lim_{x\to 0} \frac{\sin(2x)}{2x}\right)\left(\lim_{x\to 0} \frac{1}{\cos(2x)}\right) = 2$$

5. (*20 Points*): The height (in meters) of a projectile shot vertically upward from a point 2 meters above ground level with an initial velocity of 24.5 m/s is $h = 2 + 24.5t - 4.9t^2$ after t seconds.

   (a) Find the velocity after 2 seconds and after 4 seconds.

   Solution: $v(t) = 24.5 - 9.8t$, $v(2) = 4.9$ and $v(4) = -14.7$.

   (b) When does the projectile reach its maximum height?

   Solution: When $v(t) = 0$, at $t = 2.5$ sec.

   (c) What is the maximum height?

   Solution: $h(2.5) = 32.625$ m

   (d) When does it hit the ground?

   Solution: When $h(t) = 0$, at $t = \frac{-24.5 \pm \sqrt{24.5^2 - 4(-4.9)(2)}}{-9.8} = \frac{-24.5\pm\sqrt{639.45}}{-9.8} \approx -0.08034169545549169, 5.080341695455491$. So at $t = 5.080341695455491$ sec.

   (e) With what velocity does it hit the ground?

   Solution: $v(5.080341695455491) = -25.28734861546382$ m/s

6. (*10 Points*): The altitude of a triangle is increasing at a rate of 1 cm/min while the area of the triangle is increasing at a rate of 2 cm^2/min. At what rate is the base of the triangle changing when the altitude is 10 cm and the area is 100 cm^2?

   Solution: $A = \frac{1}{2}bh$, so $\frac{dA}{dt} = \frac{1}{2}\left(h\frac{db}{dt} + b\frac{dh}{dt}\right)$. At the point of interest, $A = 100$, $h = 10$ and hence $b = 20$.

   So $2 = \frac{1}{2}\left(10\frac{db}{dt} + 20 \cdot 1\right) = 5\frac{db}{dt} + 10$, and hence $\frac{db}{dt} = -\frac{8}{5} = -1.6$ cm/min.