1. (*15 Points*): Find dy given $y = \sqrt{3+x^2}$, at $x = 1$ and $dx = -0.1$.

   Solution:
$$f'(x) = y' = \frac{1}{2}(3+x^2)^{-1/2} \cdot 2x = \frac{x}{\sqrt{3+x^2}}$$
   so $dy = f'(x)\,dx = f'(1) \cdot (-0.1) = \frac{-0.1}{2} = -0.05$

2. (*20 Points*): Find the absolute maximum and absolute minimum of the function $f(x) = 3x^4 - 4x^3 - 12x^2 + 1$ on the interval $[-2, 3]$.

   Solution:
$$f'(x) = 12x^3 - 12x^2 - 24x = 12x\,(x-2)\,(x+1)$$
   Solving $f'(x) = 0$ gives $x = 2$, $x = -1$, and $x = 0$, also $f'(x)$ exists everywhere so we do not get any critical numbers in this case. Putting each of these and the endpoints back into the original function gives,
$$\begin{aligned} f(-2) &= 33 \leftarrow \text{Absolute Maximum} \\ f(3) &= 28 \\ f(0) &= 1 \\ f(2) &= -31 \leftarrow \text{Absolute Minimum} \\ f(-1) &= -4 \end{aligned}$$

3. (*20 Points*): Verify that the function $f(x) = 2x^2 - 3x + 1$ satisfies the hypotheses of the Mean Value Theorem on the interval $[0, 2]$. Then find all numbers c that satisfy the conclusion of the Mean Value Theorem.

   Solution: $f(x) = 2x^2 - 3x + 1$ is continuous and differentiable everywhere and hence on the given interval.
$$m = \frac{f(2) - f(0)}{2 - 0} = 1 \qquad \text{and} \qquad f'(x) = 4x - 3$$
   Solving $f'(x) = 1$ gives $x = 1$, so there is only one value $c = 1$ that satisfies the conclusion of the Mean Value Theorem.

4. (*15 Points*): Find the following limit,
$$\lim_{x \to 3} \frac{\ln(x/3)}{x-3}$$
   Solution:
$$\lim_{x \to 3} \frac{\ln(x/3)}{x-3} = \lim_{x \to 3} \frac{\frac{1}{x}}{1} = \frac{1}{3}$$

5. (*40 Points*): Given the function $f(x) = 3x^4 - 4x^3 + 2$,

(a) Find $f'(x)$.
Solution: $f'(x) = 12x^3 - 12x^2 = 12x^2(x-1)$

(b) Find all the critical numbers to the function, keep your answers in exact form.
Solution: 0 and 1.

(c) Find the intervals of increasing and decreasing of the function.
Solution: Increasing on $(1, \infty)$, and decreasing on $(-\infty, 1)$,

(d) Find all local maximums and minimums of the function.
Solution: Local minimum at $x = 1$, the point $(1, 1)$.

(e) Find $f''(x)$.
Solution: $f''(x) = 36x^2 - 24x = 12x(3x-2)$

(f) Find all the places where the function could change concavity, keep your answers in exact form.
Solution: 0 and $\frac{2}{3}$

(g) Find the intervals of concave up and concave down of the function.
Solution: Concave up on $(-\infty, 0) \cup (2/3, \infty)$, and concave down on $(0, 2/3)$,

(h) Find all of the points of inflection.
Solution: Inflection points at $x = 0$ and $x = 2/3$.