Exam #2 Key

1. (*20 Points*) Do one and only one of the following integrals.

   (a) $\displaystyle \int \frac{1}{x\sqrt{4-x^2}}\, dx = -\frac{1}{2}\ln\left|\frac{\sqrt{4-x^2}}{x} + \frac{2}{x}\right| + C$

   (b) $\displaystyle \int \frac{\sqrt{9+x^2}}{x}\, dx = \sqrt{9+x^2} + \frac{3}{2}\ln\left|\frac{\sqrt{9+x^2}}{3} - 1\right| - \frac{3}{2}\ln\left|\frac{\sqrt{9+x^2}}{3} + 1\right| + C$

2. (*20 Points*) Do one and only one of the following integrals.

   (a) $\displaystyle \int \sin^2(3x)\cos^3(3x)\, dx = \frac{1}{9}\sin^3(3x) - \frac{1}{15}\sin^5(3x) + C$

   (b) $\displaystyle \int 7\tan(x)\sec^3(x)\, dx = \frac{7}{3}\sec^3(x) + C$

3. (*20 Points*) Do one and only one of the following integrals.

   (a) $\displaystyle \int 9x\cos(8x)\, dx$

   (b) $\displaystyle \int x^2\ln(x)\, dx$

4. (*20 Points*) Do one and only one of the following integrals.

   (a) $\displaystyle \int \frac{x-1}{(x+2)(x-3)^2}\, dx$

   (b) $\displaystyle \int \frac{x^2+x}{(x+5)(x^2+1)}\, dx$

5. (*20 Points*) Do one and only one of the following integrals. Determine whether the integral is convergent or divergent. If it is convergent, evaluate it.

   (a) $\displaystyle \int_{-\infty}^{\infty} xe^{-x^2}\, dx$

   (b) $\displaystyle \int_{2}^{\infty} e^{-7x}\, dx$

6. (*10 Points*) This set of exercises all deal with the integral,

   $$\int_0^4 \sin(x\cos(x))\, dx$$

   (a) Use Simpson's Rule with $n = 4$ to approximate the integral. Your answer must be correct to at least 5 decimal places. **Solution:** $S_4 = -0.20247603324799155735$

   (b) Find the error bound for this approximation. Your answer must be correct to at least 5 decimal places. The graphs of the second and fourth derivatives of $f(x) = \sin(x\cos(x))$ respectively are below.
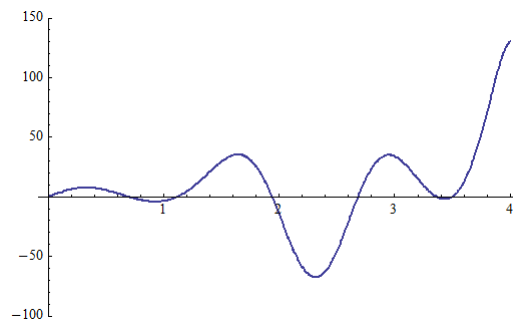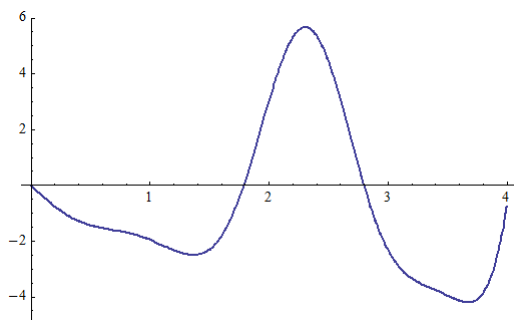
   Solution: $\displaystyle |E_S| \le \frac{K(b-a)^5}{180n^4} \le \frac{150\cdot4^5}{180\cdot4^4} = \frac{10}{3} \approx 3.3333333333333333333$