Fall 2013 **Final Exam** Math 202

Name: _____

Write all of your responses on these exam pages. If you need more space please use the backs. Make sure that you show all of your work, answers without supporting work will receive no credit.

1. (*15 Points*) The following questions deal with the integral,

$$\int_0^2 x^2 + x - 1 \, dx$$

(a) Approximate the integral by using a right-hand Riemann sum with 4 rectangles.

(b) Set up the integral as a limit of a right-hand Riemann sum.

(c) Evaluate the integral by evaluating the limit of the right-hand Riemann sum, leave your answer in exact form. Recall the following summation formulas we went over in class.

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2} \qquad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \qquad \sum_{i=1}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2$$

2. (*10 Points Each*) Find the following integrals, for all definite integrals leave your final answers in exact form.

(a) $\displaystyle\int \frac{dx}{\cos^2(x)\sqrt{1+\tan(x)}}$

(b) $\displaystyle\int \frac{x^3}{\sqrt{9-x^2}}\,dx$

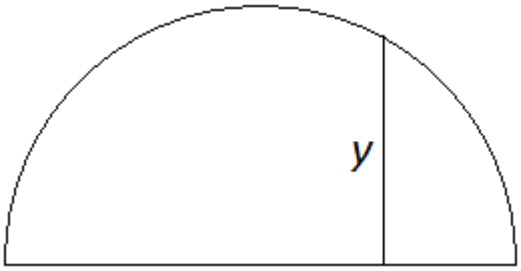
(c) $\displaystyle\int \frac{x^2 - x + 6}{x^3 + 3x}\, dx$

(d) $\displaystyle\int_0^{2\pi} x^2 \sin(3x)\, dx$

(e) $\displaystyle\int \cos^3(x) \sin(2x)\, dx$

(f) $\displaystyle\int_{-\infty}^{\infty} \frac{e^x}{e^{2x}+4}\,dx$

3. (*15 Points*) Find the volume of the object that has a semi-circular base of radius 2 and perpendicular cross sections to the diameter of the semi-circle are squares. In the image below, the length y is the length of the side of the square.



4. (*15 Points*) Write the integral that will find the surface area of the curve $y = \ln(1 + x)$ rotated about the y-axis over the interval $0 \leq x \leq 1$ but do not evaluate the integral. Write a Mathematica command that will compute this integral.

5. (*15 Points Each*) Determine if the following series converge or diverge, show all of the details to the tests you perform.

(a) $\displaystyle\sum_{n=1}^{\infty}(-1)^n\frac{\sqrt{n}}{2n+3}$

(b) $\displaystyle\sum_{n=2}^{\infty}\frac{1}{n(\ln(n))^3}$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{n+1}{n^2\sqrt{n+2}}$

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{10^n}{(n+1)4^{2n+1}}$

6. (*15 Points*) Find the radius and interval of convergence of the following power series.

$$\sum_{n=1}^{\infty} \frac{(x-2)^n}{\sqrt{n^2+1}}$$

7. (*15 Points*) Find the Taylor series for $f(x) = \dfrac{1}{x^2}$ centered at $x = 2$, write your answer in summation notation and determine its interval of convergence of the series.

8. (*15 Points*) Find the solution to the differential equation $y' \cos^2(x) = y + 1$ with the initial condition of $y(\pi/4) = 1$. Write your final answer in $y = f(x)$ form.