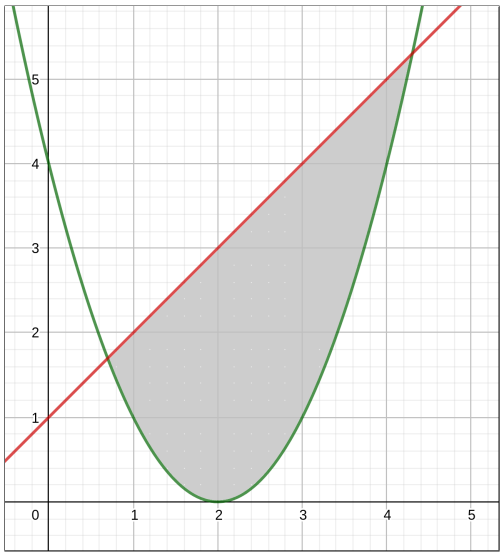
Name: _____

Write all of your responses on these exam pages, if you need extra space please use the backs of the pages. Show all your work, answers without supporting justification will not receive credit. Keep your answers in exact form and simplify numerical answers to the point where a scientific calculator could get an approximation. Each exercise is worth 20 points. **No calculation devices allowed.**

1. (*10 Points*) Set up, but do not evaluate, the integrals for finding the volume of the solid obtained by rotating the region between $y = (x-2)^2$ and $y = x+1$ about,

   (a) The x-axis.

   (b) The y-axis.



Solution:

(a) The x-axis.

$$\int_{\frac{5-\sqrt{13}}{2}}^{\frac{5+\sqrt{13}}{2}} \pi(x+1)^2 - \pi(x-2)^4 \, dx$$

(b) The y-axis.

$$\int_{\frac{5-\sqrt{13}}{2}}^{\frac{5+\sqrt{13}}{2}} 2\pi x \left(x+1-(x-2)^2\right) \, dx$$

2. (*15 Points*) Find the following integral,

$$\int \frac{x^2}{\sqrt{1-x^2}}\,dx$$

Solution: Using $x = \sin(\theta)$

$$\begin{aligned}
\int \frac{x^2}{\sqrt{1-x^2}}\,dx &= \int \frac{\sin^2(\theta)}{\cos(\theta)}\cos(\theta)\,d\theta \\
&= \int \sin^2(\theta)\,d\theta \\
&= \frac{1}{2}\int 1-\cos(2\theta)\,d\theta \\
&= \frac{1}{2}\theta - \frac{1}{4}\sin(2\theta)\,d\theta \\
&= \frac{1}{2}\theta - \frac{1}{2}\sin(\theta)\cos(\theta) \\
&= \frac{1}{2}\sin^{-1}(x) - \frac{1}{2}x\sqrt{1-x^2} + C
\end{aligned}$$

3. (*15 Points*) Find the following integral,

$$\int \frac{1}{x^2-4x+3}\,dx$$

Solution:

$$\int \frac{1}{x^2-4x+3}\,dx = \frac{1}{2}\int \frac{1}{x-3}\,dx - \frac{1}{2}\int \frac{1}{x-1}\,dx = \frac{1}{2}\ln|x-3| - \frac{1}{2}\ln|x-1| + C$$

4. (*15 Points*) Find the following integral,

$$\int \tan^3(x)\sec(x)\,dx$$

Solution:

$$\begin{aligned}
\int \tan^3(x)\sec(x)\,dx &= \int \tan^2(x)\tan(x)\sec(x)\,dx \\
&= \int (\sec^2(x)-1)\tan(x)\sec(x)\,dx \\
&= \int u^2 - 1\,du \\
&= \frac{u^3}{3} - u + C \\
&= \frac{\sec^3(x)}{3} - \sec(x) + C
\end{aligned}$$

5. (*15 Points*) Find the following integral,

$$\int x\cos(x)\,dx$$

Solution:

$$\int x\cos(x)\,dx = x\sin(x) - \int \sin(x)\,dx = x\sin(x) + \cos(x) + C$$

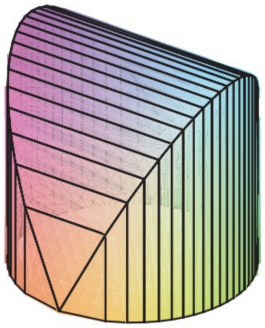
6. (*15 Points*) Find the following integral,

$$\int \frac{\cos^3(x)}{\csc(x)}\,dx$$

Solution:

$$\int \frac{\cos^3(x)}{\csc(x)}\,dx = \int \cos^3(x)\sin(x)\,dx = -\int u^3\,du = -\frac{1}{4}u^4 + C = -\frac{1}{4}\cos^4(x) + C$$

7. (*15 Points*) Find the volume of the object that has a circular base with radius r and the parallel cross-sections perpendicular to a diameter of the base are squares.



Solution: Cross-section area is $A(x) = 4(r^2 - x^2)$, so

$$V = 4\int_{-r}^{r} r^2 - x^2\,dx = 4(r^2x - r^3/3)\Big|_{-r}^{r} = \frac{16}{3}r^3$$