1. Express the integral as a limit of Riemann sums using left endpoints. Then use the properties of sums and limits to evaluate the integral, do not use the Fundamental Theorem of Calculus.

$$\int_1^3 3x^2 + 2x \, dx$$

Solution: 34

2. Given that

$$F(x) = \int_x^{x^2} e^{t^2} \, dt$$

find $F'(x)$.

Solution: $2xe^{x^4} - e^{x^2}$

3. Evaluate the following integral

$$\int \frac{\sin(x)\cos(x)}{1+\sin^2(x)} \, dx$$

Solution:

$$\frac{\ln\left(\sin^2(x)+1\right)}{2} + C$$

4. Evaluate the following integral

$$\int_1^2 x^2 \ln(x) \, dx$$

Solution:

$$\frac{x^3 \ln(x)}{3} - \frac{x^3}{9} \qquad \frac{8\ln(2)}{3} - \frac{7}{9}$$

5. Evaluate the following integral

$$\int x \cos^5(x^2) \, dx$$

Solution:

$$\frac{1}{2}\sin(x^2) - \frac{1}{3}\sin^3(x^2) + \frac{1}{10}\sin^5(x^2) + C$$

6. Evaluate the following integral

$$\int x\sqrt{1-x^4} \, dx$$

Solution:

$$\frac{1}{4}\sin^{-1}(x^2) + \frac{1}{4}x^2\sqrt{1-x^4} + C$$

7. Evaluate the following integral

$$\int \frac{x-4}{x^2-5x+6}\,dx$$

Solution:

$$2\ln(x-2)-\ln(x-3)$$

8. Evaluate the following integral

$$\int_1^\infty \frac{1}{x^2+x}\,dx$$

Solution: $\ln(2)$

9. Test the series for convergence or divergence.

$$\sum_{n=1}^{\infty} \frac{3^n n^2}{n!}$$

Solution: Converges by the ratio test.

10. Test the series for convergence or divergence.

$$\sum_{n=1}^{\infty} n^2 e^{-n^3}$$

Solution: Converges by the integral test.

11. Test the series for convergence or divergence.

$$\sum_{n=1}^{\infty} \frac{1}{n\sqrt{n^2+1}}$$

Solution: Converges by direct comparison to $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2}$.

12. Find the radius of convergence and interval of convergence of the power series.

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{(2n-1)2^n}(x-1)^n$$

Solution: The radius of convergence is 2 and the interval of convergence is $(-1,3]$.

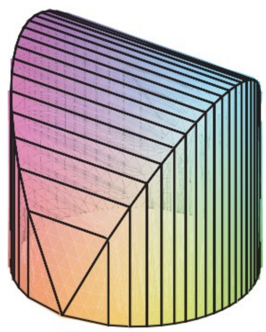
13. Find the Taylor series for $f(x) = \dfrac{1}{x^2}$ centered at $a = 1$, and find the associated radius of convergence. You may assume that f has a power series expansion and you need not verify that $R_n(x) \to 0$.

Solution:

$$f(x) = \frac{1}{x^2} = \sum_{n=0}^{\infty} (-1)^n (n+1)(x-1)^n$$

The associated radius of convergence is 1.

14. Find the volume of the object that has a circular base with radius r and the parallel cross-sections perpendicular to a diameter of the base are squares.



Solution: Cross-section area is $A(x) = 4(r^2 - x^2)$, so

$$V = 4\int_{-r}^{r} r^2 - x^2 \, dx = 4(r^2 x - r^3/3)\Big|_{-r}^{r} = \frac{16}{3} r^3$$