1. (*15 points*) Use the limit of the Riemann sum definition with the right-hand endpoint to find the following integral. Evaluate the integral keeping your answer in exact form.

$$\int_{-1}^{2} 4x^2 + x + 2 \, dx$$

Solution: $\Delta x = \frac{3}{n}$, $x_i = a + i\Delta x = -1 + \frac{3i}{n}$. So

$$\begin{aligned}
\int_{-1}^{2} 4x^2 + x + 2 \, dx &= \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\,\Delta x \\
&= \lim_{n\to\infty} \sum_{i=1}^{n} \left(4\left(-1 + \frac{3i}{n}\right)^2 + \left(-1 + \frac{3i}{n}\right) + 2 \right) \frac{3}{n} \\
&= \lim_{n\to\infty} \sum_{i=1}^{n} \frac{15}{n} - \frac{63i}{n^2} + \frac{108i^2}{n^3} \\
&= \lim_{n\to\infty} \frac{15}{n} \cdot n - \frac{63}{n^2} \cdot \frac{n(n+1)}{2} + \frac{108}{n^3} \cdot \frac{n(n+1)(2n+1)}{6} \\
&= \lim_{n\to\infty} 15 - \frac{63}{n^2} \cdot \frac{n^2+n}{2} + \frac{108}{n^3} \cdot \frac{2n^3 + \cdots}{6} = 15 - \frac{63}{2} + 36 = \frac{39}{2}
\end{aligned}$$

2. (*10 points*) Using the Fundamental Theorem of Calculus, find

$$\frac{d}{dx} \int_{\sin(x)}^{100} \tan(t^2) \, dt$$

Solution: $-\tan(\sin^2(x))\cos(x)$

3. (*15 points*) Using the Fundamental Theorem of Calculus, find

(a) $\displaystyle\int_0^4 (4-x)\sqrt[3]{x} \, dx$

Solution:

$$\begin{aligned}
\int_0^4 (4-x)\sqrt[3]{x} \, dx = \int_0^4 4x^{1/3} - x^{4/3} \, dx &= 3x^{4/3} - \frac{3}{7}x^{7/3} \Big|_0^4 \\
&= 3 \cdot 4^{4/3} - \frac{3}{7} \cdot 4^{7/3} = 3 \cdot 4^{4/3} - \frac{3}{7} \cdot 4^{7/3} \\
&= 12\sqrt[3]{4} - \frac{48\sqrt[3]{4}}{7} = \frac{36\sqrt[3]{4}}{7}
\end{aligned}$$

(b) $\displaystyle\int_0^1 e^x + x^e \, dx$

Solution:

$$\int_0^1 e^x + x^e \, dx = e^x + \frac{x^{e+1}}{e+1}\Big|_0^1 = e + \frac{1}{e+1} - 1 = \frac{e^2}{e+1}$$

4. (*10 points*) Find the following indefinite integrals

(a) $\displaystyle\int x^2 + 1 + \frac{1}{x^2+1}\, dx$

Solution:
$$\int x^2 + 1 + \frac{1}{x^2+1} = \frac{x^3}{3} + x + \tan^{-1}(x) + C$$

(b) $\displaystyle\int \sin(x)\sqrt{1+\cos(x)}\, dx$

Solution: Let $u = 1 + \cos(x)$, then $du = -\sin(x)\, dx$ and $dx = \frac{du}{-\sin(x)}$
$$\begin{aligned}\int \sin(x)\sqrt{1+\cos(x)}\, dx &= \int \sin(x)\sqrt{u}\frac{du}{-\sin(x)} = -\int \sqrt{u}\, du = -\int u^{1/2}\, du \\ &= -\frac{2}{3}u^{3/2} + C = -\frac{2}{3}(1+\cos(x))^{3/2} + C\end{aligned}$$

5. **Extra Credit:** (*5 points*) Use areas (that is of standard geometric figures) to calculate the following. Keep your answer in exact form and simplify when possible. Draw pictures of the region(s) being evaluated.
$$\int_{3/2}^{4} 3 + \sqrt{4 + 3x - x^2}\, dx$$

Solution: First integral is a rectangle and the second is a quarter of a circle with center $(3/2, 0)$ and radius $5/2$.
$$\begin{aligned}\int_{3/2}^{4} 3 + \sqrt{4 + 3x - x^2}\, dx &= \int_{3/2}^{4} 3\, dx + \int_{3/2}^{4} \sqrt{4 + 3x - x^2}\, dx \\ &= 3 \cdot \frac{5}{2} + \int_{3/2}^{4} \sqrt{25/4 - (x - 3/2)^2}\, dx \\ &= \frac{15}{2} + \frac{1}{4}\pi(5/2)^2 = \frac{15}{2} + \frac{25\pi}{16}\end{aligned}$$