1. (*15 points*) Use the limit of the Riemann sum definition with the right-hand endpoint to find the following integral. Evaluate the integral keeping your answer in exact form.

$$\int_1^5 4 - 7x + x^2 \, dx$$

Solution: $\Delta x = \frac{4}{n}$, $x_i = a + i\Delta x = 1 + \frac{4i}{n}$. So

$$\begin{aligned}
\int_1^5 4 - 7x + x^2 \, dx &= \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\,\Delta x \\
&= \lim_{n\to\infty} \sum_{i=1}^{n} \left(4 - 7\left(1 + \frac{4i}{n}\right) + \left(1 + \frac{4i}{n}\right)^2\right) \frac{4}{n} \\
&= \lim_{n\to\infty} \sum_{i=1}^{n} -\frac{8}{n} - \frac{80i}{n^2} + \frac{64i^2}{n^3} \\
&= \lim_{n\to\infty} -\frac{8}{n} \cdot n - \frac{80}{n^2} \cdot \frac{n(n+1)}{2} + \frac{64}{n^3} \cdot \frac{n(n+1)(2n+1)}{6} \\
&= -8 - 40 + \frac{64}{3} = -\frac{80}{3}
\end{aligned}$$

2. (*10 points*) Using the Fundamental Theorem of Calculus, find

$$\frac{d}{dx} \int_5^{x^2-4x} \ln(t^2)\, dt$$

Solution: $\ln((x^2 - 4x)^2)(2x - 4)$

3. (*15 points*) Using the Fundamental Theorem of Calculus, find

(a) $\displaystyle\int_1^2 (4 + x^2)^3 \, dx$

Solution:

$$\begin{aligned}
\int_1^2 (4 + x^2)^3 \, dx &= \int_1^2 x^6 + 12x^4 + 48x^2 + 64 \, dx \\
&= \frac{x^7}{7} + \frac{12x^5}{5} + 16x^3 + 64x \Bigg|_1^2 \\
&= \left(\frac{2^7}{7} + \frac{12 \cdot 2^5}{5} + 16 \cdot 2^3 + 128\right) - \left(\frac{1}{7} + \frac{12}{5} + 16 + 64\right) = \frac{9399}{35}
\end{aligned}$$

(b) $\displaystyle\int_1^5 \frac{3x^2 + 1}{x^3} \, dx$

Solution:

$$\begin{aligned}
\int_1^5 \frac{3x^2 + 1}{x^3} \, dx &= \int_1^5 \frac{3}{x} + \frac{1}{x^3} \, dx = 3\ln|x| - \frac{1}{2x^2} \Bigg|_1^5 \\
&= \left(3\ln(5) - \frac{1}{50}\right) - \left(3\ln(1) - \frac{1}{2}\right) = 3\ln(5) + \frac{12}{25}
\end{aligned}$$

4. (*10 points*) Find the following indefinite integrals

   (a) $\int \sqrt[3]{x} + \frac{1}{\sqrt[3]{x}}\, dx$

   Solution:

$$\int \sqrt[3]{x} + \frac{1}{\sqrt[3]{x}}\, dx = \int x^{1/3} + x^{-1/3}\, dx = \frac{3x^{4/3}}{4} + \frac{3x^{2/3}}{2} + C$$

   (b) $\int \frac{\sin(1/x)}{x^2}\, dx$

   Solution: Let $u = 1/x$, then $du = -1/x^2\, dx$ and $dx = -x^2\, du$

$$\int \frac{\sin(1/x)}{x^2}\, dx = \int -x^2 \frac{\sin(u)}{x^2}\, du = -\int \sin(u)\, du = \cos(u) + C = \cos(1/x) + C$$

5. **Extra Credit:** (*5 points*) Use areas (that is of standard geometric figures) to calculate the following. Keep your answer in exact form and simplify when possible. Draw pictures of the region(s) being evaluated.

$$\int_a^b 3x + \sqrt{20 - x^2}\, dx$$

   where a and b are the bounds on the domain of the integrand. Find the exact values of a and b and calculate the integral using areas.

   Solution: The values of a and b are $-\sqrt{20}$ and $\sqrt{20}$. First integral is a difference of two triangles with the same area (net 0) and the second is half of a circle with center $(0, 0)$ and radius $\sqrt{20}$.

$$\int_{-\sqrt{20}}^{\sqrt{20}} 3x + \sqrt{20 - x^2}\, dx = \int_{-\sqrt{20}}^{\sqrt{20}} 3x\, dx + \int_{-\sqrt{20}}^{\sqrt{20}} \sqrt{20 - x^2}\, dx$$
$$= \frac{\pi \cdot (\sqrt{20})^2}{2} = 10\pi$$