1. (*25 Points*): Find the following integral,

$$\int \frac{x+2}{x^2+3x-4}\,dx$$

Solution:

$$\int \frac{x+2}{x^2+3x-4}\,dx = \int \frac{2}{5\,(x+4)} + \frac{3}{5\,(x-1)}\,dx = \frac{3}{5}\ln|x-1| + \frac{2}{5}\ln|x+4| + C$$

2. (*25 Points*): Find the following integral,

$$\int x\cos^2(x)\,dx$$

Solution: Let $u = x$ and $dv = \cos^2(x)\,dx$, then $du = dx$ and $v = \frac{1}{2}x + \frac{1}{4}\sin(2x)$. So

$$\begin{aligned}
\int x\cos^2(x)\,dx &= x\left(\frac{1}{2}x + \frac{1}{4}\sin(2x)\right) - \int \frac{1}{2}x + \frac{1}{4}\sin(2x)\,dx \\
&= x\left(\frac{1}{2}x + \frac{1}{4}\sin(2x)\right) - \left(\frac{1}{4}x^2 - \frac{1}{8}\cos(2x)\right) + C \\
&= \frac{x^2}{4} + \frac{x\sin(2x)}{4} + \frac{\cos(2x)}{8} + C
\end{aligned}$$

3. (*25 Points*): Find the following integral,

$$\int \frac{1}{x\sqrt{4x^2+1}}\,dx$$

Solution: Let $x = \frac{1}{2}\tan(\theta)$, then $dx = \frac{1}{2}\sec^2(\theta)\,d\theta$, and

$$\begin{aligned}
\int \frac{1}{x\sqrt{4x^2+1}}\,dx &= \int \frac{1}{\frac{1}{2}\tan(\theta)\sqrt{\tan^2(\theta)+1}} \cdot \frac{1}{2}\sec^2(\theta)\,d\theta \\
&= \int \frac{1}{\tan(\theta)\sec(\theta)} \cdot \sec^2(\theta)\,d\theta \\
&= \int \frac{\sec(\theta)}{\tan(\theta)}\,d\theta = \int \csc(\theta)\,d\theta = -\ln|\csc(\theta)+\cot(\theta)| + C \\
&= -\ln\left|\frac{\sqrt{4x^2+1}}{2x} + \frac{1}{2x}\right| + C
\end{aligned}$$

4. (*25 Points*): Find the following integral,

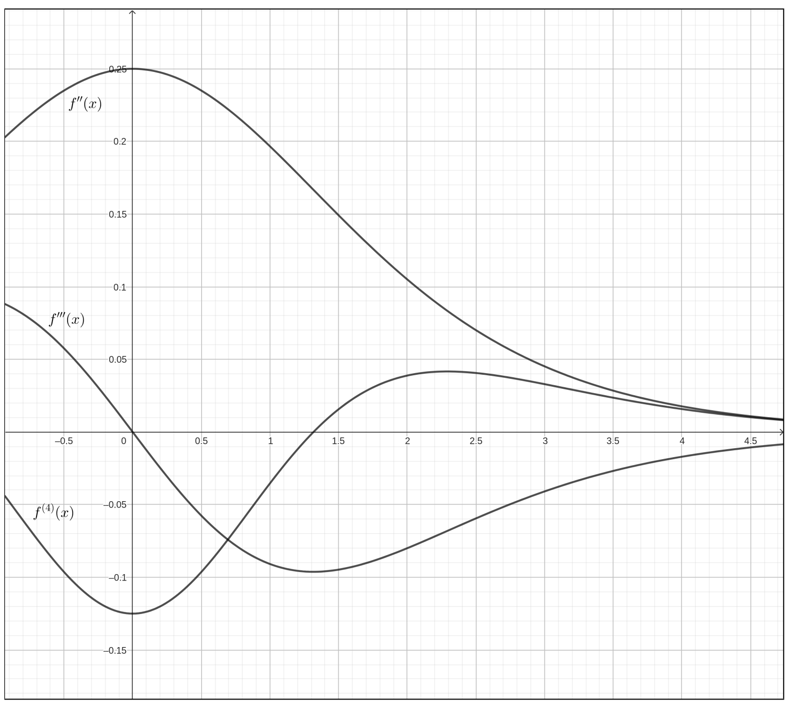
$$\int \frac{1}{\sqrt{x+1}+\sqrt{x}}\,dx$$

Solution:

$$\int \frac{1}{\sqrt{x+1}+\sqrt{x}}\,dx = \int \frac{1}{\sqrt{x+1}+\sqrt{x}} \cdot \frac{\sqrt{x+1}-\sqrt{x}}{\sqrt{x+1}-\sqrt{x}}\,dx = \int \sqrt{x+1}-\sqrt{x}\,dx$$
$$= \frac{2}{3}(x+1)^{3/2} - \frac{2}{3}x^{3/2} + C$$

5. (*10 Points*): Use (a) the Trapezoidal Rule, (b) the Midpoint Rule, and (c) Simpson's Rule to approximate the given integral with $n = 4$ subdivisions. Also determine the error bounds for each of the three methods. The graphs of $f''(x)$, $f'''(x)$, and $f^{(4)}(x)$ are given below. Give your answers to at least 5 significant digits.

$$\int_0^4 \ln(1+e^x)\,dx$$



Solution: Let $f(x) = \ln(1+e^x)$, $\Delta x = \frac{b-a}{n} = \frac{4-0}{4} = 1$

(a) $T = \frac{1}{2}(f(0)+2f(1)+2f(2)+2f(3)+f(4)) \approx 8.844425604373814$
$|E_T| \leq \frac{K(b-a)^3}{12n^2} \leq \frac{0.25\cdot 4^3}{12\cdot 4^2} \approx 0.08333333333333334$

(b) $M = f(0.5)+f(1.5)+f(2.5)+f(3.5) \approx 8.78413041472803$
$|E_M| \leq \frac{K(b-a)^3}{24n^2} \leq \frac{0.25\cdot 4^3}{24\cdot 4^2} \approx 0.0416666666666667$

(c) $S = \frac{1}{3}(f(0)+4f(1)+2f(2)+4f(3)+f(4)) \approx 8.804183095643852$
$|E_S| \leq \frac{K(b-a)^5}{180n^4} \leq \frac{0.125\cdot 4^5}{180\cdot 4^4} \approx 0.002777777777777778$