Name: _____

Write all of your responses on this quiz paper, use the back if necessary. Show all your work, answers without supporting justification will not receive credit.

1. (*25 Points*): Determine whether the integral

$$\int_1^\infty \frac{1}{x^2 + x} \, dx$$

   is convergent or divergent. If it is convergent, find its value.

2. (*25 Points*): Find the exact length of the curve,

$$f(x) = \frac{x^2}{4} - \frac{\ln(x)}{2}$$

on $1 \leq x \leq 2$.

3. (*25 Points*): Find the exact area of the surface obtained by rotating the curve $y^2 = x+1$, $0 \le x \le 3$, about the x-axis.

4. (*25 Points*): Determine whether the sequence

$$a_n = n^2 e^{-n}$$

converges or diverges. If it converges, find the limit.

5. **Extra Credit**: (*10 Points*): **Do one and only one of the following.**

(a) A sequence $\{a_n\}$ is defined by $a_1 = \sqrt{2}$ and $a_{n+1} = \sqrt{2 + a_n}$. That is, the sequence

   is $\sqrt{2},\ \sqrt{2 + \sqrt{2}},\ \sqrt{2 + \sqrt{2 + \sqrt{2}}},\ \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2}}}},\ \ldots$.

   i. Show that $\{a_n\}$ is increasing and bounded above by 3, hence it converges.

   ii. Find $\lim_{n \to \infty} a_n$.

(b) **Gabriel's Horn**: The surface formed by rotating the curve $y = 1/x$, $x \geq 1$, about the x-axis is known as Gabriel's horn. Show that the surface area is infinite and show that the volume is finite. Hence this is an object you can fill with paint but you cannot paint it.