Spring 2012 # Exam #1 Math 306

Name: _____

Write all of your responses on these exam pages. If you need more space please use the backs. Make sure that you show all of your work.

1. **Definitions:** (*4 Points Each*) Give a definition for each of the following.

   (a) The Reduced Row Echelon form of a Matrix

   (b) A Linear Combination of a Set of Vectors

   (c) The Span of a Set of Vectors

   (d) A Linear Transformation

   (e) Linear Independence and Dependence

2. **True and False:** (*3 Points Each*) Mark each of the following as either true or false. If the statement is false either give a counterexample or correct the statement so that it is true.

(a) _____ The solution set to a linear system involving variables $x_1, \ldots, x_n$ is a list of numbers $(s_1, \ldots, s_n)$ that makes each equation in the system a true statement when the values $s_1, \ldots, s_n$ are substituted for $x_1, \ldots, x_n$ respectively.

(b) _____ If the solution set to $A\mathbf{x} = \mathbf{0}$ is a plane through the origin then the solution set to $A\mathbf{x} = \mathbf{b}$ is a translation of that plane (assuming $\mathbf{b} \neq \mathbf{0}$).

(c) _____ Elementary row operations on an augmented matrix never change the solution set of the associated linear system.

(d) _____ Elementary row operations can change the pivot positions.

(e) _____ The row echelon form of a matrix is unique.

(f) _____ The span of two non-zero vectors in $\mathbb{R}^3$ is a plane through the origin.

(g) _____ $A\mathbf{x}$ is a linear combination of the columns of A.

(h) _____ If A is an $m \times n$ matrix whose columns span $\mathbb{R}^m$ then for each vector $\mathbf{b} \in \mathbb{R}^m$ the system $A\mathbf{x} = \mathbf{b}$ has an infinite number of solutions.

(i) _____ Any subset of a linearly dependent set is dependent.

(j) _____ The composition of three linear transformations may not result in a linear transformation.

3. **Calculations:** (*10 Points Each*) Do each of the following.

   (a) Write the following system as both a matrix equation and as a vector equation. Then solve the system and put your final answer in parametric vector form. Show each step in the reduction and label the reduction step as we did in class. Finally, write the vector $(-1, -5, -17)$ as a linear combination of the columns of the matrix and describe the solution set geometrically.

$$\begin{aligned} 2x_1 + x_2 + 8x_3 &= -1 \\ 3x_1 + x_2 + 10x_3 &= -5 \\ 13x_1 + 5x_2 + 46x_3 &= -17 \end{aligned}$$

(b) Consider the linear transformation T defined by $T(x_1, x_2, x_3) = (x_1 - 2x_3, x_3 + x_2 - 4x_1)$.

i. What is the domain of T?

ii. What is the codomain of T?

iii. If A is the matrix such that $T(\mathbf{x}) = A\mathbf{x}$, what is the size of A?

iv. Find the matrix A such that $T(\mathbf{x}) = A\mathbf{x}$.

v. Is T a one-to-one map? Justify your answer.

vi. Is T an onto map? Justify your answer.

(c) For each of the following sets of vectors, tell me if the set is linearly independent or dependent and why. Do as few operations as possible to answer the question.

i. $\{(1,-2),(2,4)\}$

ii. $\{(-7,21,-14),(1,-3,2)\}$

iii. $\{(2,3,13),(1,1,5),(8,10,46)\}$

iv. $\{(-31,29,33),(41,13,125),(0,47,-17),(13,19,101)\}$

(d) For each of the following sets of vectors, describe geometrically the span of the set of vectors.

i. $\{(1,-2),(2,4)\}$

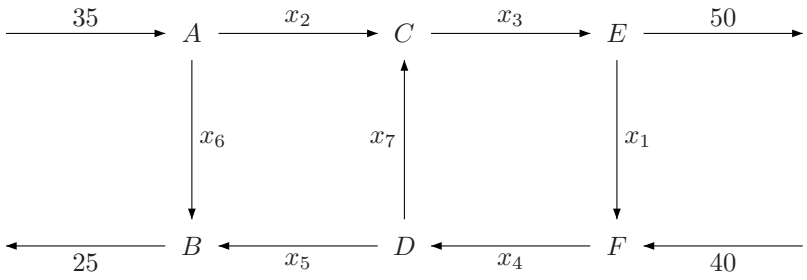
ii. $\{(-7,21,-14),(1,-3,2)\}$

iii. $\{(2,3,13),(1,1,5),(8,10,46)\}$

iv. $\{(-31,29,33),(41,13,125),(0,47,-17),(13,19,101)\}$
Note that the reduced row echelon form of the matrix having this set of vectors as its columns is,

$$\begin{bmatrix} 1 & 0 & 0 & \frac{64311}{136390} \\ 0 & 1 & 0 & \frac{91871}{136390} \\ 0 & 0 & 1 & -\frac{4978}{68195} \end{bmatrix}$$

(e) Given the following network flow, set up the matrix that describes the general flow pattern, do not solve the system.



The following is the reduced echelon form of the network flow matrix, what restrictions does this solution place on the traffic flow?

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 1 & -1 & -15 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 35 \\ 0 & 0 & 1 & 0 & 0 & 1 & -1 & 35 \\ 0 & 0 & 0 & 1 & 0 & 1 & -1 & 25 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 25 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(f) In a certain region, about 7% of a city's population moves to the surrounding suburbs each year, and about 5% of the suburban population moves into the city. In 2010, there were 800,000 residents in the city and 500,000 in the suburbs. Set up the migration matrix for this situation and estimate the populations of the city and suburbs in 2012.