Name: ____

Write all of your responses on this quiz paper, use the back if necessary. Show all your work, answers without supporting justification will not receive credit.

1. (*30 points*): Mark each of the following as either True or False by circling the correct answer.

   (a) **True** **False**: Elementary row operations on an augmented matrix never change the solution set of the associated linear system.

   (b) **True** **False**: Two matrices are row equivalent if they have the same number of rows.

   (c) **True** **False**: The pivot positions in a matrix depend on whether row interchanges are used in the row reduction process.

   (d) **True** **False**: Suppose a system of linear equations has a 3×5 augmented matrix whose fifth column is a pivot column. The system consistent?

   (e) **True** **False**: When $\mathbf{u}$ and $\mathbf{v}$ are nonzero vectors, $\text{Span}\{\mathbf{u}, \mathbf{v}\}$ contains the line through $\mathbf{u}$ and the origin.

   (f) **True** **False**: The equation $A\mathbf{x} = \mathbf{b}$ is consistent if the augmented matrix $[A\ \mathbf{b}]$ has a pivot position in every row.

   (g) **True** **False**: The equation $A\mathbf{x} = \mathbf{b}$ is homogeneous if the zero vector is a solution.

   (h) **True** **False**: The solution set of a consistent system $A\mathbf{x} = \mathbf{b}$ is obtained by translating the solution set of $A\mathbf{x} = \mathbf{0}$.

   (i) **True** **False**: The columns of the matrix A are linearly independent if the equation $A\mathbf{x} = \mathbf{0}$ has the trivial solution.

   (j) **True** **False**: The columns of any 4×5 matrix are linearly dependent.

2. (*40 points*): For the following system of linear equations,

$$\begin{aligned} x_1 + 2x_2 + 3x_3 &= 4 \\ 4x_1 + 5x_2 + 6x_3 &= 7 \\ 6x_1 + 7x_2 + 8x_3 &= 9 \end{aligned}$$

(a) Write the system in matrix equation form $A\mathbf{x} = \mathbf{b}$, explicitly denote A, $\mathbf{x}$, and $\mathbf{b}$.

(b) Construct the associated augmented matrix for the system.

(c) Reduce the augmented matrix to reduced row echelon form.

(d) Is the system consistent or inconsistent?

(e) If the system is consistent,

i. Write the solution in parametric form.

ii. Write the solution in parametric vector form.

iii. What are the basic variables and what are the free variables?

iv. Describe the set of solutions geometrically.

3. (*10 points*): Compute

$$\begin{bmatrix} 6 & 5 \\ -4 & -3 \\ 7 & 6 \end{bmatrix} \begin{bmatrix} 1 \\ -3 \end{bmatrix}$$

4. (*20 points*): Consider the following set of vectors,

$$\mathbf{v}_1 = \begin{bmatrix} 0 \\ 3 \\ -1 \\ 1 \end{bmatrix}, \ \mathbf{v}_2 = \begin{bmatrix} -8 \\ -7 \\ 5 \\ -3 \end{bmatrix}, \ \mathbf{v}_3 = \begin{bmatrix} 5 \\ 4 \\ -4 \\ 2 \end{bmatrix}, \ \mathbf{v}_4 = \begin{bmatrix} -2 \\ 5 \\ 1 \\ 1 \end{bmatrix}$$

If we form the matrix using these vectors as columns, in the order given, and reduce the matrix to reduced row echelon form we obtain,

$$\begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

(a) Does this set of vectors form an independent or dependent set?

(b) Is any vector in this set a linear combination of the others? If so write the vector as a linear combination of the other vectors. You may use $\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{v}_3$, and $\mathbf{v}_4$ instead of the actual vectors to save time.

(c) Do these vectors span $\mathbb{R}^4$? Why or why not?

5. **Extra Credit**: (*10 points*): Given the following network, write the system of linear equations and its corresponding augmented matrix that describes the general flow pattern in the network, **do not solve the system**.