Name: ____

Write all of your responses on this exam paper, use the back if necessary. Show all your work, answers without supporting justification will not receive credit.

1. (*30 points*): Mark each of the following as either True or False by circling the correct answer.

   (a) **True** **False**: If A is an $m \times n$ matrix, then the range of the transformation $\mathbf{x} \to A\mathbf{x}$ is $\mathbb{R}^m$.

   (b) **True** **False**: Every linear transformation is a matrix transformation.

   (c) **True** **False**: When two linear transformations are performed one after another, the combined effect may not always be a linear transformation.

   (d) **True** **False**: A linear transformation $T : \mathbb{R}^n \to \mathbb{R}^m$ is completely determined by its effect on the columns of the $n \times n$ identity. matrix.

   (e) **True** **False**: $AB + AC = A(B + C)$

   (f) **True** **False**: Each column of AB is a linear combination of the columns of B using weights from the corresponding column of A.

   (g) **True** **False**: If A is an invertible $n \times n$ matrix, then the equation $A\mathbf{x} = \mathbf{b}$ is consistent for *each* $\mathbf{b}$ in $\mathbb{R}^n$

   (h) **True** **False**: Each elementary matrix is invertible.

   (i) **True** **False**: If A is an $n \times n$ matrix and the columns of A span $\mathbb{R}^n$, then the columns of A are linearly independent.

   (j) **True** **False**: If A^T is not invertible, then A is not invertible.

2. (*15 points*): Find the matrix of the transformation from $\mathbb{R}^2$ to $\mathbb{R}^2$ that rotates all the vectors counterclockwise by an angle of $\theta = \frac{\pi}{3}$ and then reflects the vector through the origin. Keep your answer in exact form.

3. (*15 points*): Let A and B be as follows, compute AB and BA. If the computation is undefined state why.

$$A = \begin{bmatrix} 1 & 2 & -3 \\ 3 & 7 & 5 \end{bmatrix} \qquad B = \begin{bmatrix} 4 & -3 & 2 \\ 1 & -1 & 1 \\ 2 & 5 & -3 \end{bmatrix}$$

4. (*20 points*): Find the inverse of the following matrix if it exists. If it does not exist, state why.

$$A = \begin{bmatrix} 8 & 3 & 1 \\ -17 & -7 & -2 \\ 10 & 4 & 1 \end{bmatrix}$$

5. (*20 points*): Find the LU decomposition of the matrix,

$$A = \begin{bmatrix} 2 & -1 & 2 \\ -6 & 0 & -2 \\ 8 & -1 & 5 \end{bmatrix}$$

6. **Extra Credit**: (*10 points*): Suppose A is an $n \times n$ matrix and the equation $A\mathbf{x} = \mathbf{0}$ has only the trivial solution. Let $\mathbf{b} = \begin{bmatrix} 1 \\ 2 \\ \vdots \\ n \end{bmatrix}$ show that $A\mathbf{x} = \mathbf{b}$ has a solution.