Name: ____

Write all of your responses on this exam paper, use the back if necessary. Show all your work, answers without supporting justification will not receive credit.

1. (*30 points*): Mark each of the following as either True or False by circling the correct answer.

   (a) **True** **False**: The cofactor expansion of $\det(A)$ down a column is equal to the cofactor expansion along a row.

   (b) **True** **False**: The determinant of a triangular matrix is the sum of the entries on the main diagonal.

   (c) **True** **False**: If the columns of A are linearly dependent, then $\det(A) = 0$.

   (d) **True** **False**: Cramer's rule can only be used for invertible matrices.

   (e) **True** **False**: $\mathbb{R}^2$ is a subspace of $\mathbb{R}^3$.

   (f) **True** **False**: The polynomials of degree two or less are a subspace of the polynomials of degree three or less.

   (g) **True** **False**: The column space of A is the range of the mapping $\mathbf{x} \mapsto A\mathbf{x}$.

   (h) **True** **False**: The kernel of a linear transformation is a vector space.

   (i) **True** **False**: The null space of an $m \times n$ matrix is in $\mathbb{R}^m$.

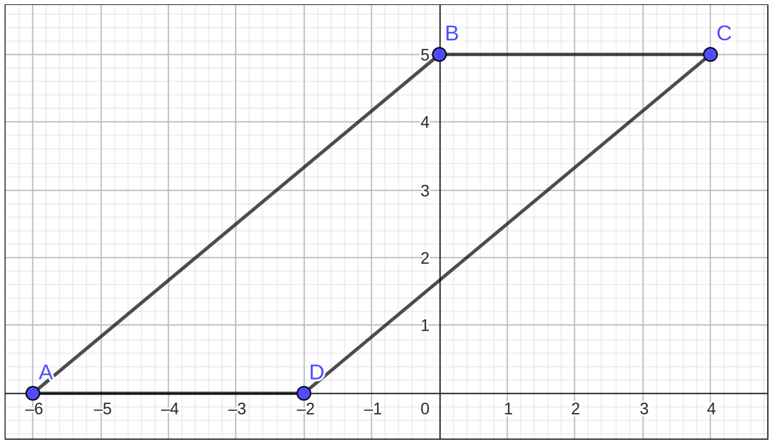
   (j) **True** **False**: The column space of an $m \times n$ matrix is in $\mathbb{R}^m$.

2. (*15 points*): Find the determinant of

$$\begin{bmatrix} 1 & 5 & 4 & 3 & 2 \\ 0 & 8 & 5 & 9 & 0 \\ 0 & 7 & 0 & 0 & 0 \\ 3 & 9 & 6 & 5 & 4 \\ 0 & 8 & 0 & 6 & 0 \end{bmatrix}$$

Is the matrix invertible?

3. (*15 points*): Find the area of the parallelogram whose vertices are $(-6, 0)$, $(0, 5)$, $(4, 5)$, and $(-2, 0)$.



4. (*15 points*): Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation given by the matrix,

$$A = \begin{bmatrix} 1 & 4 \\ 3 & 1 \end{bmatrix}$$

Let S represent the unit disk, that is the region $x^2 + y^2 \leq 1$, what is the area of $T(S)$?

5. (*15 points*): Let

$$W = \left\{ \begin{bmatrix} 4a+3b \\ 0 \\ a+b+c \\ c-2a \end{bmatrix}, a, b, c \in \mathbb{R} \right\}$$

find a set S of vectors that spans W or give an example to show that W is not a vector space.

6. (*10 points*): Let H and K be subspaces of a vector space V. Show that the intersection $K \cap H$ is a subspace of V.

7. **Extra Credit**: (*10 points*): Find the determinants of the following matrices,

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{bmatrix} \qquad \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 2 & 2 \\ 1 & 2 & 3 & 3 \\ 1 & 2 & 3 & 4 \end{bmatrix} \qquad \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 2 & 2 & 2 \\ 1 & 2 & 3 & 3 & 3 \\ 1 & 2 & 3 & 4 & 4 \\ 1 & 2 & 3 & 4 & 5 \end{bmatrix}$$

Conjecture, but you do not need to prove, what the determinant is of the matrix,

$$\begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & 2 & 2 & \cdots & 2 \\ 1 & 2 & 3 & \cdots & 3 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 2 & 3 & \cdots & n \end{bmatrix}$$