Name: ______________________________

Write all of your responses on these pages, use the backs if necessary. Show all your work, answers without supporting justification will not receive credit.

1. (*25 points*): Given the following matrix A,

$$A = \begin{bmatrix} -1 & -5 & -1 & -6 & 1 \\ 2 & 10 & 1 & 10 & -3 \\ -1 & -5 & 0 & -4 & 2 \end{bmatrix}$$

   (a) Find a basis to the column space of A.

   (b) Find a basis to the row space of A.

   (c) Find a basis to the null space of A.

2. (*25 points*): Given the following vectors, and $\mathcal{B} = \{\mathbf{b_1}, \mathbf{b_2}\}$ and $\mathcal{C} = \{\mathbf{c_1}, \mathbf{c_2}\}$

$$\mathbf{b_1} = \begin{bmatrix} 7 \\ 5 \end{bmatrix} \quad \mathbf{b_2} = \begin{bmatrix} -3 \\ -1 \end{bmatrix} \quad \mathbf{c_1} = \begin{bmatrix} 1 \\ -5 \end{bmatrix} \quad \mathbf{c_2} = \begin{bmatrix} -2 \\ 2 \end{bmatrix}$$

(a) Find $\underset{\mathcal{B} \leftarrow \mathcal{C}}{P}$

(b) Find $\underset{\mathcal{C} \leftarrow \mathcal{B}}{P}$

(c) Given that $[\mathbf{x}]_{\mathcal{B}} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$, find $[\mathbf{x}]_{\mathcal{C}}$.

3. (*50 points*): Given the following matrix,

$$A = \begin{bmatrix} 2 & 2 & -1 \\ 1 & 3 & -1 \\ -1 & -2 & 2 \end{bmatrix}$$

(a) Find the characteristic polynomial of A.

(b) Find all the eigenvalues of A, and their algebraic multiplicities. Hints: $\lambda = 1$ is an eigenvalue, and all eigenvalues to this matrix are integers.

(c) Find bases to each of the eigenspaces for A.

(d) What is the dimension of each eigenspace and what geometric object is each eigenspace?

(e) Is A diagonalizable? If so, find P and D such that $A = PDP^{-1}$.

(f) Is A invertible? Why or why not?

(g) If we define $T : \mathbb{R}^3 \to \mathbb{R}^3$ as $T(\mathbf{x}) = A\mathbf{x}$, and we let $\mathcal{B}$ be the set of eigenvectors to A, what is $[T]_{\mathcal{B}}$?

(h) Is T one-to-one? Why or why not?

(i) Is T onto? Why or why not?

(j) Is there a plane through the origin in $\mathbb{R}^3$ that T maps to a line through the origin? If so, find a plane that maps to a line, and if not, state why no such plane exists.

Note that the next page is blank for you to continue your solutions to this exercise.

4. **Extra Credit**: (*10 points*): The following matrix A was subjected to many iterations of the QR method and the result was the matrix B. Find all the eigenvalues to the matrix A.

$$A = \begin{bmatrix} -\frac{117}{11} & -\frac{54}{11} & -\frac{30}{11} & -\frac{60}{11} & -\frac{21}{11} & -\frac{30}{11} \\ -\frac{90}{11} & \frac{59}{33} & -\frac{10}{33} & \frac{46}{33} & \frac{16}{11} & \frac{56}{33} \\ -\frac{130}{11} & -\frac{455}{33} & -\frac{287}{33} & -\frac{244}{33} & -\frac{60}{11} & \frac{208}{33} \\ \frac{710}{11} & \frac{1000}{33} & \frac{712}{33} & \frac{929}{33} & \frac{150}{11} & \frac{52}{33} \\ \frac{134}{11} & \frac{50}{11} & \frac{18}{11} & \frac{58}{11} & \frac{39}{11} & \frac{40}{11} \\ -\frac{441}{11} & -\frac{878}{33} & -\frac{599}{33} & -\frac{703}{33} & -\frac{124}{11} & \frac{61}{33} \end{bmatrix} \qquad B = \begin{bmatrix} 5 & 7 & 9 & -4 & 3 & 15 \\ 0 & 2 & -1 & -1 & 1 & 6 \\ 0 & 1 & 2 & -5 & 8 & 1 \\ 0 & 0 & 0 & -3 & -2 & -1 \\ 0 & 0 & 0 & 0 & 5 & -4 \\ 0 & 0 & 0 & 0 & 4 & 5 \end{bmatrix}$$