Name: _____

Write all your work and solutions on the extra paper provided and hand it in with this exam paper. Please put your name on each page in case the pages get separated.

1. (*25 Points*): Given the two vectors $\mathbf{u} = \langle 2, 3, 4\rangle$ and $\mathbf{v} = \langle 1, -1, 2\rangle$, find
   (a) $\mathbf{u} \cdot \mathbf{v}$
   (b) $\mathbf{u} \times \mathbf{v}$
   (c) $|\mathbf{u}|$
   (d) The angle between $\mathbf{u}$ and $\mathbf{v}$.
   (e) The equation of the plane containing the points $P_1 = (3, 5, 1)$, $P_2 = (5, 8, 5)$, and $P_3 = (4, 4, 3)$. Hint: $\overrightarrow{P_1P_2} = \mathbf{u}$ and $\overrightarrow{P_1P_3} = \mathbf{v}$.

2. (*25 Points*): Given the space curve $\mathbf{r}(t) = \mathbf{i} + t^2\mathbf{j} + t^3\mathbf{k}$
   (a) Set up the integral for finding the length of this curve for $0 \leq t \leq 1$.
   (b) Evaluate the integral to find the length of this curve for $0 \leq t \leq 1$.

3. (*20 Points*): Given the space curve,
$$\mathbf{r}(t) = \left\langle te^{-t}, \frac{t^3 + t}{2t^3 - 1}, t\sin\left(\frac{1}{t}\right)\right\rangle$$
   (a) Find $\lim_{t\to\infty} \mathbf{r}(t)$, if it exists.
   (b) Find $\mathbf{r}'(t)$.

4. (*30 Points*): Given the space curve $\mathbf{r}(t) = \langle 3\sin(t), 2t, 3\cos(t)\rangle$, find $\mathbf{T}(t)$, $\mathbf{N}(t)$, and the curvature.

5. (*10 Points Extra Credit*): Find the equation for the torsion of $\mathbf{r}(t) = \langle 3\sin(t), 2t, 3\cos(t)\rangle$.