Name: _____

Write all your work and solutions on the extra paper provided and hand it in with this exam paper. Please put your name on each page in case the pages get separated.

1. (*15 Points*): Find the limit, if it exists, or show that the limit does not exist.

$$\lim_{(x,y)\to(0,0)} \frac{xy^3}{x^2+y^6}$$

2. (*20 Points*): Do one and only one of the following:

   (a) Find all the first and second partial derivatives of $f(x,y) = e^{-2x}\cos(y^2)$.

   (b) Find an equation of the tangent plane to the surface $z = x\sin(x+y)$ at the point $(-1,1)$.

3. (*20 Points*): Use the Chain Rule to find $\partial z/\partial s$ and $\partial z/\partial t$ when $z = \ln(3x+2y)$, $x = s\sin(t)$, and $y = t\cos(s)$.

4. (*20 Points*): Find the directional derivative of $z = x^2e^{-y}$ at $(3,0)$ in the direction $\mathbf{u} = \langle 3,4\rangle$. In what direction, on this surface and at that point, would the directional derivative be a maximum?

5. (*25 Points*): Do one and only one of the following:

   (a) Find the absolute maximum and minimum values of

$$f(x,y) = x^2 + xy + y^2 - 6y$$

   on the domain $D = \{(x,y) | -3 \le x \le 3, 0 \le y \le 5\}$.

   (b) Find the local maximum and minimum values and saddle point(s) of the function,

$$f(x,y) = x^3 + y^3 - 3x^2 - 3y^2 - 9y$$

   (c) Use Lagrange multipliers to find the extreme values of the function $f(x,y) = xy$ subject to the constraint $4x^2 + y^2 = 8$.

6. (*10 Points*): Do either of the two exercises you did not do in problem #5. Do only one of them.