1. (*20 Points*): Set up **and evaluate** the double integral of $f(x,y) = x^2\sin^3(y)$ over the domain $D = [0,3]\times[0,\pi/2]$.

   Solution:

$$\begin{aligned}
\int_0^{\pi/2}\int_0^3 x^2\sin^3(y)\,dx\,dy &= \left(\int_0^3 x^2\,dx\right)\left(\int_0^{\pi/2}\sin^3(y)\,dy\right)\\
&= \left(\frac{x^3}{3}\bigg|_0^3\right)\left(\int_0^{\pi/2}\sin(y)(1-\cos^2(y))\,dy\right)\\
&= 9\left(\int u^2-1\,du\right)\\
&= 9\left(\frac{1}{3}\cos^3(y)-\cos(y)\bigg|_0^{\pi/2}\right)\\
&= 9\cdot\frac{2}{3} = 6
\end{aligned}$$

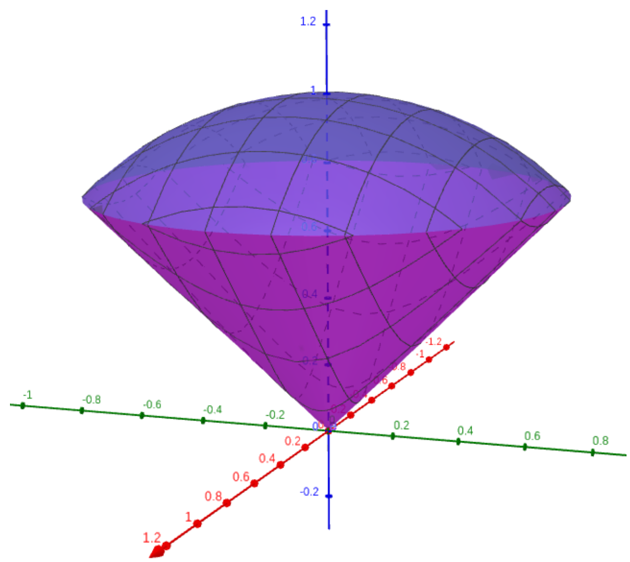
2. (*15 Points*): Set up **but do not evaluate** the double integral that will find the volume under the surface $f(x,y) = xy$ and above the triangle with vertices $(1,1)$, $(4,1)$, and $(1,2)$.

   Solution:

$$\int_1^4\int_1^{-1/3x+7/3} xy\,dy\,dx$$

3. (*15 Points*): Set up **but do not evaluate** the integral that will find the volume of the object above the cone $z=\sqrt{x^2+y^2}$ and below the sphere $x^2+y^2+z^2=1$. Write the integral using polar coordinates. Image of the object is below.

   Solution:



The intersection of the sphere and the cone (projected onto the xy-plane) is the circle $x^2 + y^2 = 1/2$. So in rectangular coordinates the integral would be,

$$\int_{-\sqrt{2}/2}^{\sqrt{2}/2} \int_{-\sqrt{1/2-x^2}}^{\sqrt{1/2-x^2}} \sqrt{1-(x^2+y^2)} - \sqrt{x^2+y^2}\, dy\, dx$$

and in polar coordinates we get

$$\int_0^{2\pi} \int_0^{\sqrt{2}/2} \left(\sqrt{1-r^2} - r\right) r\, dr\, d\theta$$

4. (*15 Points*): Set up **but do not evaluate** the integrals that will find the center of mass $(\bar{x}, \bar{y})$ of the lamina that occupies the region enclosed by the curves $y = 0$ and $y = \cos(x)$, with $-\pi/2 \le x \le \pi/2$ and density function $\rho(x, y) = x^2 e^{-y}$.

   Solution: $\bar{x} = M_y/m$ and $\bar{y} = M_x/m$ with

$$\begin{aligned} m &= \int_{-\pi/2}^{\pi/2} \int_0^{\cos(x)} x^2 e^{-y}\, dy\, dx \\ M_x &= \int_{-\pi/2}^{\pi/2} \int_0^{\cos(x)} x^2 y e^{-y}\, dy\, dx \\ M_y &= \int_{-\pi/2}^{\pi/2} \int_0^{\cos(x)} x^3 e^{-y}\, dy\, dx \end{aligned}$$

5. (*15 Points*): Set up **but do not evaluate** the integral that will find the surface area of the part of the surface $z = 1 + x^2 y^2$ that lies above the disk $x^2 + y^2 \le 1$.

   (a) Set this up in rectangular coordinates.

   (b) Set this up in polar coordinates.

   Solution:

   (a) In rectangular coordinates.

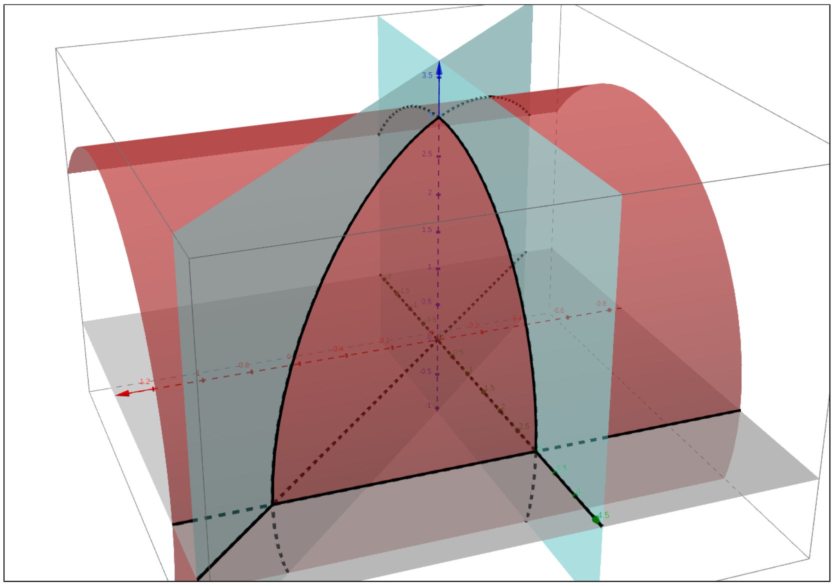
$$S = \iint_D \sqrt{f_x^2 + f_y^2 + 1}\, dA = \int_{-1}^{1} \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \sqrt{4x^2y^4 + 4x^4y^2 + 1}\, dy\, dx$$

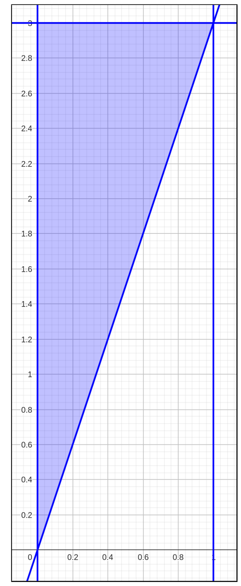
   (b) In polar coordinates.

$$\begin{aligned} S &= \int_{-1}^{1} \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \sqrt{4x^2y^4 + 4x^4y^2 + 1}\, dy\, dx \\ &= \int_0^{2\pi} \int_0^1 \sqrt{4r^2\cos^2(\theta) r^4 \sin^4(\theta) + 4r^4 \cos^4(\theta)^4 r^2 \sin^2(\theta) + 1}\, r\, dr\, d\theta \\ &= \int_0^{2\pi} \int_0^1 r\sqrt{4r^6 \cos^2(\theta)\sin^2(\theta) + 1}\, dr\, d\theta \end{aligned}$$

6. (*20 Points*): Set up **and evaluate** $\iiint_E z\ dV$ where E is bounded by the cylinder $y^2+z^2=9$ and the planes $x=0$, $y=3x$, and $z=0$ in the first octant. Image of E is below, the first octant is facing you.

Solution: The region E and the domain from the xy-plane that we are integrating over.





$$\begin{aligned}
\iiint\limits_E z\ dV &= \int_0^1 \int_{3x}^3 \int_0^{\sqrt{9-y^2}} z\ dz\ dy\ dx \\
&= \int_0^1 \int_{3x}^3 \left. \frac{z^2}{2} \right|_0^{\sqrt{9-y^2}} dy\ dx \\
&= \frac{1}{2}\int_0^1 \int_{3x}^3 9-y^2\ dy\ dx \\
&= \frac{1}{2}\int_0^1 9y - \left.\frac{y^3}{3}\right|_{3x}^3 dx \\
&= \frac{1}{2}\int_0^1 (27-9)-(27x-9x^3)\ dx \\
&= \frac{1}{2}\int_0^1 9x^3-27x+18\ dx \\
&= \frac{1}{2}\left(\frac{9}{4}x^4 - \frac{27}{2}x^2 + 18x\Big|_0^1\right) \\
&= \frac{1}{2}\left(\frac{9}{4} - \frac{27}{2} + 18\right) = \frac{27}{8}
\end{aligned}$$

7. **Extra Credit** (*10 Points*): Do the integral from #2 or #3, just do one of them.

Solution:

$$\begin{aligned}
\int_1^4 \int_1^{-1/3x+7/3} xy\ dy\ dx &= \frac{1}{2}\int_1^4 xy^2\Big|_1^{-1/3x+7/3}\ dx \\
&= \frac{1}{2}\int_1^4 x\left(-\frac{1}{3}x+\frac{7}{3}\right)^2 - x\ dx \\
&= \frac{1}{18}\int_1^4 x^3 - 14x^2 + 40x\ dx = \frac{31}{8}
\end{aligned}$$

$$\begin{aligned}
\int_0^{2\pi}\int_0^{\sqrt{2}/2}\left(\sqrt{1-r^2}-r\right) r\ dr\ d\theta &= \int_0^{2\pi}\int_0^{\sqrt{2}/2} r\sqrt{1-r^2} - r^2\ dr\ d\theta \\
&= \int_0^{2\pi} d\theta \int_0^{\sqrt{2}/2} r\sqrt{1-r^2} - r^2\ dr \\
&= 2\pi \int_0^{\sqrt{2}/2} r\sqrt{1-r^2} - r^2\ dr \\
&= 2\pi\left(-\frac{1}{3}(1-r^2)^{3/2} - \frac{r^3}{3}\Big|_0^{\sqrt{2}/2}\right) \\
&= \frac{2\pi}{3}\left(-(1-r^2)^{3/2} - r^3\Big|_0^{\sqrt{2}/2}\right) \\
&= \frac{2\pi}{3}\left(-\frac{1}{2^{3/2}} - \frac{1}{2^{3/2}} + 1\right) = \frac{\pi}{3}\left(2-\sqrt{2}\right)
\end{aligned}$$