Name: ______________________________

Write all your work and solutions on the extra paper provided and hand it in with this exam paper. Please put your name on each page in case the pages get separated. Each exercise is worth 20 points.

1. Given $\mathbf{a} = \langle 1, 1, -2 \rangle$ and $\mathbf{b} = \langle 3, -2, 1 \rangle$, find
   (a) $2\mathbf{a} + 3\mathbf{b}$ (b) $|\mathbf{b}|$ (c) $\mathbf{a} \cdot \mathbf{b}$ (d) $\mathbf{a} \times \mathbf{b}$ (e) $\text{proj}_{\mathbf{a}}\mathbf{b}$
   (f) The angle between **a** and **b** (correct to the nearest degree)

2. Find an equation of the plane through the line of intersection of the planes $x - z = 1$ and $y + 2z = 3$ and perpendicular to the plane $x + y - 2z = 1$.

3. Find the length of the curve $\mathbf{r}(t) = \langle 2t^{3/2}, \cos(2t), \sin(2t) \rangle$, $0 \le t \le 1$.

4. Do one and only one of the following,
   (a) Find the curvature of the ellipse $x = 3\cos(t)$, $y = 4\sin(t)$ at the points $(3, 0)$ and $(0, 4)$.
   (b) Find the unit tangent, normal, and binormal vectors for $\mathbf{r}(t) = \langle \sqrt{2}t, e^t, e^{-t} \rangle$ at the point $(0, 1, 1)$.

5. Find the local maximum and minimum values and saddle points of the function $f(x, y) = 3xy - x^2y - xy^2$.

6. Do one and only one of the following,
   (a) Use Lagrange multipliers to find the maximum and minimum values of $f(x, y, z) = xyz$ subject to the constraint $x^2 + y^2 + z^2 = 3$.
   (b) Find equations of the tangent plane and the normal line to $\sin(xyz) = x + 2y + 3z$ at the point $(2, -1, 0)$.

7. Do one and only one of the following,
   (a) Find
   $$\iint_D (x^2 + y^2)^{3/2}\, dA$$
   where D is the region in the first quadrant bounded by the lines $y = 0$, $y = \sqrt{3}x$, and the circle $x^2 + y^2 = 9$.
   (b) Evaluate the integral
   $$\int_0^1 \int_x^1 \cos(y^2)\, dy\, dx$$

8. Use spherical coordinates to evaluate
   $$\int_{-2}^{2} \int_0^{\sqrt{4-y^2}} \int_{-\sqrt{4-x^2-y^2}}^{\sqrt{4-x^2-y^2}} y^2 \sqrt{x^2 + y^2 + z^2}\, dz\, dx\, dy$$

9. Do one and only one of the following,

   (a) Evaluate the line integral $\int_C \sqrt{xy}\, dx + e^y\, dy + xz\, dz$ where C is given by $\mathbf{r}(t) = \langle t^4, t^2, t^3 \rangle$ for $0 \leq t \leq 1$.

   (b) Show that $\mathbf{F}(x, y) = (4x^3y^2 - 2xy^3)\,\mathbf{i} + (2x^4y - 3x^2y^2 + 4y^3)\,\mathbf{j}$ is conservative and use that fact to find $\int_C \mathbf{F} \cdot d\mathbf{r}$ where $C : \mathbf{r}(t) = (t + \sin(\pi t))\,\mathbf{i} + (2t + \cos(\pi t))\,\mathbf{j}$, $0 \leq t \leq 1$.

10. Use Green's Theorem to evaluate $\int_C \sqrt{1 + x^3}\, dx + 2xy\, dy$ where C is the triangle with vertices $(0, 0)$, $(1, 0)$, and $(1, 3)$.

Extra Credit: (*10 points*): Evaluate the surface integral $\iint_S \mathbf{F} \cdot d\mathbf{S}$ where $\mathbf{F}(x, y, z) = xz\,\mathbf{i} - 2y\,\mathbf{j} + 3x\,\mathbf{k}$ and S is the sphere $x^2 + y^2 + z^2 = 4$ with outward orientation.