1 String Manipulation and Read/Write Head Movement

The examples here are all assuming that $\Sigma = \{a, b\}$ and that $\Gamma = \Sigma \cup \{\Box\}$. Similar transition sets can be created for other alphabets. In these examples, the state q_f represents the final and hence halting state.

1. The Turing machine that will move right until it hits a blank, $R_\Box$.

$$\begin{aligned}\delta(q_0, a) &= (q_0, a, R)\\ \delta(q_0, b) &= (q_0, b, R)\\ \delta(q_0, \Box) &= (q_1, \Box, R)\\ \delta(q_1, a) &= (q_f, a, L)\\ \delta(q_1, b) &= (q_f, b, L)\\ \delta(q_1, \Box) &= (q_f, \Box, L)\end{aligned}$$

2. The Turing machine that will move left until it hits a blank, $L_\Box$.

$$\begin{aligned}\delta(q_0, a) &= (q_0, a, L)\\ \delta(q_0, b) &= (q_0, b, L)\\ \delta(q_0, \Box) &= (q_1, \Box, L)\\ \delta(q_1, a) &= (q_f, a, R)\\ \delta(q_1, b) &= (q_f, b, R)\\ \delta(q_1, \Box) &= (q_f, \Box, R)\end{aligned}$$

3. The Turing machine that will move right until it hits an a, R_a.

$$\begin{aligned}\delta(q_0, a) &= (q_1, a, R)\\ \delta(q_0, b) &= (q_0, b, R)\\ \delta(q_0, \Box) &= (q_0, \Box, R)\\ \delta(q_1, a) &= (q_f, a, L)\\ \delta(q_1, b) &= (q_f, b, L)\\ \delta(q_1, \Box) &= (q_f, \Box, L)\end{aligned}$$

4. The Turing machine that will move left until it hits an a, L_a.

$$\begin{aligned}\delta(q_0, a) &= (q_1, a, L)\\ \delta(q_0, b) &= (q_0, b, L)\\ \delta(q_0, \Box) &= (q_0, \Box, L)\\ \delta(q_1, a) &= (q_f, a, R)\\ \delta(q_1, b) &= (q_f, b, R)\\ \delta(q_1, \Box) &= (q_f, \Box, R)\end{aligned}$$

5. Turing machines that move in one direction until a particular tape character is encountered are defined similarly, for example R_b and L_b.

6. The Turing machine that will move right until it hits a non-blank symbol, $R_{\overline{\Box}}$.

$$\begin{aligned}\delta(q_0, a) &= (q_1, a, R)\\ \delta(q_0, b) &= (q_1, b, R)\\ \delta(q_0, \Box) &= (q_0, \Box, R)\\ \delta(q_1, a) &= (q_f, a, L)\\ \delta(q_1, b) &= (q_f, b, L)\\ \delta(q_1, \Box) &= (q_f, \Box, L)\end{aligned}$$

7. The Turing machine that will move right until it hits a non-a symbol, $R_{\overline{a}}$.

$$\begin{aligned}\delta(q_0, a) &= (q_0, a, R)\\ \delta(q_0, b) &= (q_1, b, R)\\ \delta(q_0, \Box) &= (q_1, \Box, R)\\ \delta(q_1, a) &= (q_f, a, L)\\ \delta(q_1, b) &= (q_f, b, L)\\ \delta(q_1, \Box) &= (q_f, \Box, L)\end{aligned}$$

8. Turing machines that move in one direction until they hit a character that is not a particular character are defined similarly, for example $R_{\overline{b}}$, $L_{\overline{\Box}}$, $L_{\overline{a}}$, and $L_{\overline{b}}$.

9. The Turing Machine that will shift a word one cell to the left, Shl. It is assumed that the character to the left of the word is a blank. It is also assumed that the read/write head is on the first character of the word to be shifted.

$$\begin{aligned}\delta(q_0, a) &= (q_a, \Box, L)\\ \delta(q_0, b) &= (q_b, \Box, L)\\ \delta(q_a, \Box) &= (q_r, a, R)\\ \delta(q_b, \Box) &= (q_r, b, R)\\ \delta(q_r, \Box) &= (q_0, \Box, R)\\ \delta(q_0, \Box) &= (q_f, \Box, L)\end{aligned}$$

10. The Turing Machine that will shift a word one cell to the right, Shr. It is assumed that the character to the right of the word is a blank. It is also assumed that the read/write head is on the last character of the word to be shifted.

$$\begin{aligned}
\delta(q_0, a) &= (q_a, \Box, R)\\
\delta(q_0, b) &= (q_b, \Box, R)\\
\delta(q_a, \Box) &= (q_r, a, L)\\
\delta(q_b, \Box) &= (q_r, b, L)\\
\delta(q_r, \Box) &= (q_0, \Box, L)\\
\delta(q_0, \Box) &= (q_f, \Box, R)
\end{aligned}$$

11. The Turing Machine that will copy a word w, Cp. Specifically, given w on the tape with the read/write head on the first letter of the word, the machine will produce $w\Box w$ on the tape. It is assumed that there are only blanks after w on the tape when the machine starts.

$$\begin{aligned}
\delta(q_0, a) &= (q_1, x, R)\\
\delta(q_1, a) &= (q_1, a, R)\\
\delta(q_1, b) &= (q_1, b, R)\\
\delta(q_1, \Box) &= (q_2, \Box, R)\\
\delta(q_2, a) &= (q_2, a, R)\\
\delta(q_2, b) &= (q_2, b, R)\\
\delta(q_2, \Box) &= (q_r, a, L)\\
\delta(q_0, b) &= (q_3, y, R)\\
\delta(q_3, a) &= (q_3, a, R)\\
\delta(q_3, b) &= (q_3, b, R)\\
\delta(q_3, \Box) &= (q_4, \Box, R)\\
\delta(q_4, a) &= (q_4, a, R)\\
\delta(q_4, b) &= (q_4, b, R)\\
\delta(q_4, \Box) &= (q_r, b, L)\\
\delta(q_r, a) &= (q_r, a, L)\\
\delta(q_r, b) &= (q_r, b, L)\\
\delta(q_r, \Box) &= (q_r, \Box, L)\\
\delta(q_r, x) &= (q_0, x, R)\\
\delta(q_r, y) &= (q_0, y, R)\\
\delta(q_0, \Box) &= (q_5, \Box, L)\\
\delta(q_5, x) &= (q_5, a, L)\\
\delta(q_5, y) &= (q_5, b, L)\\
\delta(q_5, \Box) &= (q_f, \Box, R)
\end{aligned}$$

12. It should be fairly clear how one can combine the Cp machine with the movement and Shl machines to create a Turing Machine that will take a given word w on the tape with the read/write head on the first letter of the word, the machine will produce ww on the tape. We will call this machine $CpNS$ It is assumed that there are only blanks after w on the tape when the machine starts.

13. In addition to these we can define very simple machines, such as, L and R that simply move the reading head one cell to the left or right respectively.

$$\begin{aligned}
\delta(q_0, a) &= (q_f, a, L)\\
\delta(q_0, b) &= (q_f, b, L)\\
\delta(q_0, \Box) &= (q_f, \Box, L)
\end{aligned}$$

and

$$\begin{aligned}
\delta(q_0, a) &= (q_f, a, R)\\
\delta(q_0, b) &= (q_f, b, R)\\
\delta(q_0, \Box) &= (q_f, \Box, R)
\end{aligned}$$

14. A Turing machine that will erase all letters to the right up to a space, E_R. Here we are assuming that $\Sigma = \{0, 1\}$ and that $\Gamma = \Sigma \cup \{\Box\}$. Similar transition sets can be created for other alphabets.

$$\begin{aligned}
\delta(q_0, 0) &= (q_0, \Box, R)\\
\delta(q_0, 1) &= (q_0, \Box, R)\\
\delta(q_0, \Box) &= (q_f, \Box, R)
\end{aligned}$$

15. A Turing machine that will erase all letters to the left up to a space, E_L. Here we are assuming that $\Sigma = \{0, 1\}$ and that $\Gamma = \Sigma \cup \{\Box\}$. Similar transition sets can be created for other alphabets.

$$\begin{aligned}
\delta(q_0, 0) &= (q_0, \Box, L)\\
\delta(q_0, 1) &= (q_0, \Box, L)\\
\delta(q_0, \Box) &= (q_f, \Box, L)
\end{aligned}$$

2 Arithmetic Turing Machines on Unary and Binary Numbers

The examples here are all assuming that $\Sigma = \{0, 1\}$ and that $\Gamma = \Sigma \cup \{\square\}$. The state q_f represents the final and hence halting state.

1. The Turing machine that will take a binary number on the tape and add one to it. We will call this machine A. It assumed that there is a number on the tape and that the read write head is on the far right digit of the number.

$$\begin{aligned}
\delta(q_0, 0) &= (q_1, 1, R)\\
\delta(q_0, 1) &= (q_0, 0, L)\\
\delta(q_0, \square) &= (q_2, 1, R)\\
\delta(q_1, 0) &= (q_1, 0, R)\\
\delta(q_1, 1) &= (q_1, 1, R)\\
\delta(q_1, \square) &= (q_f, \square, L)\\
\delta(q_2, 0) &= (q_2, 0, R)\\
\delta(q_2, 1) &= (q_2, 1, R)\\
\delta(q_2, \square) &= (q_f, \square, L)
\end{aligned}$$

2. The Turing machine that will take a binary number on the tape and subtract one to it. We will call this machine S. It assumed that there is a number on the tape, greater than 0, and that the read write head is on the far right digit of the number.

$$\begin{aligned}
\delta(q_0, 0) &= (q_0, 1, L)\\
\delta(q_0, 1) &= (q_1, 0, L)\\
\delta(q_1, 0) &= (q_1, 0, L)\\
\delta(q_1, 1) &= (q_1, 1, L)\\
\delta(q_1, \square) &= (q_2, \square, R)\\
\delta(q_2, 0) &= (q_3, \square, R)\\
\delta(q_2, 1) &= (q_3, 1, R)\\
\delta(q_3, 0) &= (q_3, 0, R)\\
\delta(q_3, 1) &= (q_3, 1, R)\\
\delta(q_3, \square) &= (q_f, \square, L)
\end{aligned}$$

3. A Turing machine that will compute the function $f(x) = \lfloor \frac{x}{2} \rfloor$, where x is on the tape in binary form and there is a blank before and after x.

$$\begin{aligned}
\delta(q_0, 1) &= (q_0, 1, R)\\
\delta(q_0, 0) &= (q_0, 0, R)\\
\delta(q_0, \square) &= (q_1, \square, L)\\
\delta(q_1, 0) &= (q_f, \square, L)\\
\delta(q_1, 1) &= (q_f, \square, L)
\end{aligned}$$

4. A Turing machine that will compute the function $f(x) = \lfloor \frac{x}{2} \rfloor$, where x is on the tape in unary form and there is a blank before and after x. Here we are assuming that $\Sigma = \{1\}$ and that $\Gamma = \Sigma \cup \{\square, x\}$.

$$\begin{aligned}
\delta(q_0, 1) &= (q_1, \square, R)\\
\delta(q_0, x) &= (q_4, 1, R)\\
\delta(q_1, 1) &= (q_1, 1, R)\\
\delta(q_1, \square) &= (q_2, \square, L)\\
\delta(q_1, x) &= (q_2, x, L)\\
\delta(q_2, 1) &= (q_3, x, L)\\
\delta(q_2, \square) &= (q_4, \square, R)\\
\delta(q_3, 1) &= (q_3, 1, L)\\
\delta(q_3, \square) &= (q_0, \square, R)\\
\delta(q_4, x) &= (q_4, 1, R)\\
\delta(q_4, \square) &= (q_f, \square, L)
\end{aligned}$$

3 Turing Machine Examples for Language Acceptance

The examples here are all assuming that $\Sigma = \{a, b\}$ and that $\Gamma = \Sigma \cup \{\square, x, y, z\}$. Similar transition sets can be created for other alphabets. In these examples, the state q_f represents the final and hence halting state.

1. A Turing machine that will accept the language, $L = \{a^n b^m a^{n+m} \mid n \geq 0, m \geq 1\}$.

$$
\begin{aligned}
\delta(q_0, a) &= (q_1, x, R) \\
\delta(q_0, b) &= (q_4, y, R) \\
\delta(q_1, a) &= (q_1, a, R) \\
\delta(q_1, b) &= (q_2, b, R) \\
\delta(q_2, b) &= (q_2, b, R) \\
\delta(q_2, z) &= (q_2, z, R) \\
\delta(q_2, a) &= (q_3, z, L) \\
\delta(q_3, b) &= (q_3, b, L) \\
\delta(q_3, a) &= (q_3, a, L) \\
\delta(q_3, x) &= (q_0, x, R) \\
\delta(q_3, z) &= (q_3, z, L) \\
\delta(q_4, b) &= (q_4, b, R) \\
\delta(q_4, z) &= (q_4, z, R) \\
\delta(q_4, a) &= (q_5, z, L) \\
\delta(q_5, z) &= (q_5, z, L) \\
\delta(q_5, b) &= (q_5, b, L) \\
\delta(q_5, y) &= (q_6, y, R) \\
\delta(q_6, b) &= (q_4, y, R) \\
\delta(q_6, z) &= (q_7, z, R) \\
\delta(q_7, z) &= (q_7, z, R) \\
\delta(q_7, \square) &= (q_f, \square, L)
\end{aligned}
$$

2. A Turing machine that will accept the language, $L = \{a^n b^n c^n \mid n \geq 1\}$.

$$
\begin{aligned}
\delta(q_0, a) &= (q_1, x, R) \\
\delta(q_0, y) &= (q_3, y, R) \\
\delta(q_1, a) &= (q_1, a, R) \\
\delta(q_1, y) &= (q_1, y, R) \\
\delta(q_1, b) &= (q_2, y, R) \\
\delta(q_2, b) &= (q_2, b, R) \\
\delta(q_2, z) &= (q_2, z, R) \\
\delta(q_2, c) &= (q_r, z, L) \\
\delta(q_r, b) &= (q_r, b, L) \\
\delta(q_r, y) &= (q_r, y, L) \\
\delta(q_r, a) &= (q_r, a, L) \\
\delta(q_r, z) &= (q_r, z, L) \\
\delta(q_r, x) &= (q_0, x, R) \\
\delta(q_3, y) &= (q_3, y, R) \\
\delta(q_3, z) &= (q_3, z, R) \\
\delta(q_3, \square) &= (q_f, \square, L)
\end{aligned}
$$