Implementation of Improved Parallel Stable Matching Algorithms

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ABSTRACT

Based on applications in network communications and the general matching of two sets, the stable matching problem is a topic of growing importance with the increase in availability of high-performance computing methods. The initial propositions of a sequential algorithm with a time complexity of $O(n^2)$ is too slow for modern standards. Parallel explorations of the problem have led to solutions such as the Parallel Iterative Improvement algorithm (PII) and the Convergent PII (CPII) algorithm, which present simulation-based results. Therefore, a model of implementation is warranted to verify logically-proven estimations, for ease of application and for potential relations to further stable matching considerations.

STABLE MARRIAGE PROBLEM

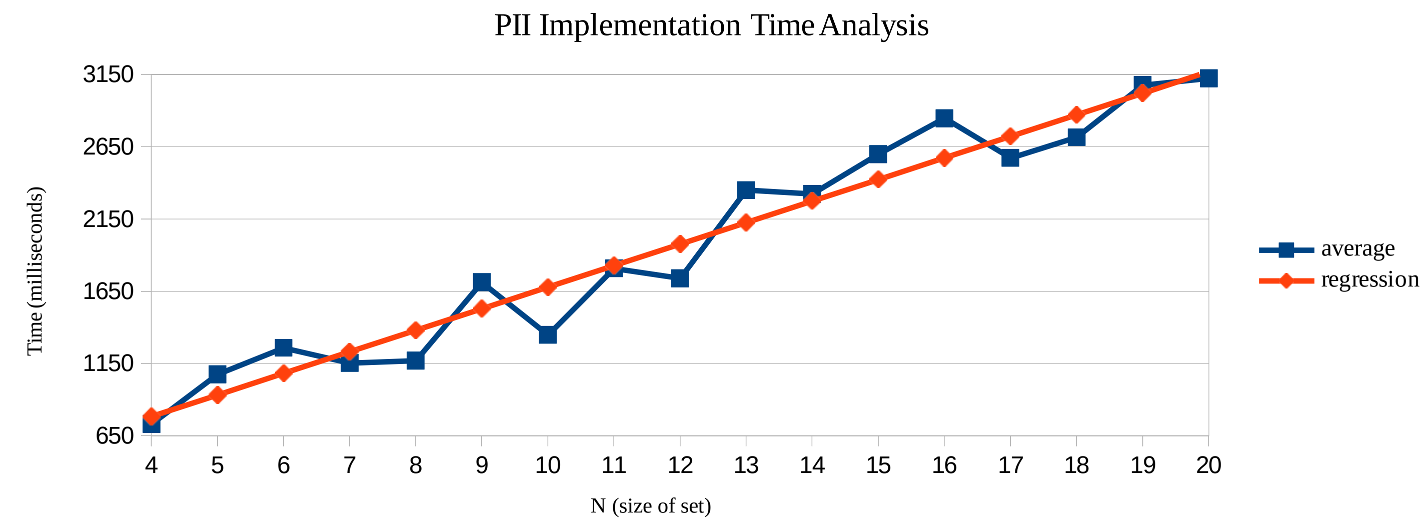
Given a set of *n* men and *n* women where each person generates a marital preference ranking list of the members of the opposite gender, the stable marriage problem seeks to find a stable matching between the two sets. A matching is stable if there exist no pairs of men and women who mutually prefer each other over their current partner. Alongside the problem proposition by Gale and Shapley, a sequential algorithm (GS algorithm) was proposed and proven functional with a $O(n^2)$ time complexity and a guarantee to produce a stable matching.

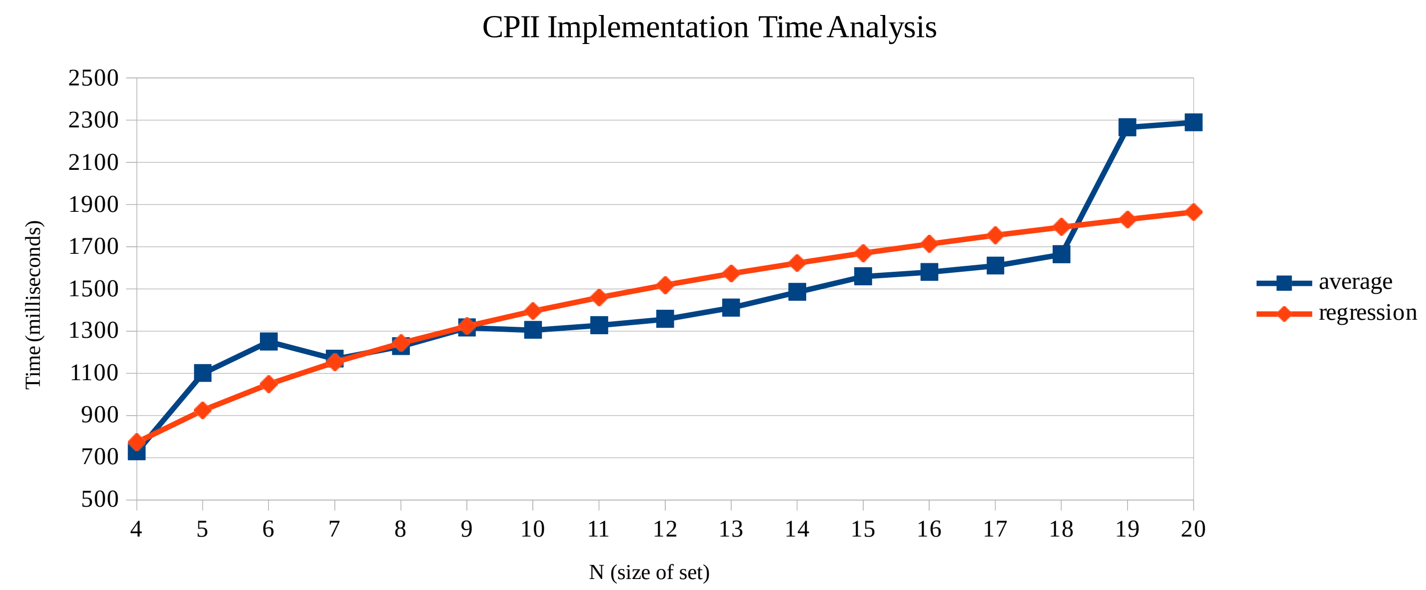
IMPLEMENTATION METHODS

Utilized technology includes OpenMPI in C++ for parallel coding and a high performance computing cluster with a maximum of four hundred processing elements (PEs). Since the PII algorithm utilizes n^2 PEs, sets of size twenty are the largest tested. Parallel implementation follows a fully connected mesh of trees topology based on message passing to separate rows and columns using communicators.

PARALLEL ITERATIVE IMPROVEMENT (PII)

The Parallel Iterative Improvement algorithm utilizes a ranking matrix of preference lists where each matrix position holds a male-female preference pairing. By using n^2 processing elements (PEs), implementation mimics the ranking matrix. The algorithm itself generates an arbitrary initial matching, identifies unstable pairings (pairings where a man and woman mutually prefer each other over their current partner) and improves upon the initial matching while considering unstable pairings. This process is repeated until a matching has no unstable pairings.





CONVERGENT PARALLEL ITERATIVE IMPROVEMENT (CPII)

The Convergent Parallel Iterative Improvement algorithm utilizes the matrix structure set by the PII to form a stable matching from initially single individual's and their preference lists. The CPII claims a guaranteed stable matching unlike the PII's 90% accuracy with cycling.

RESULTS

Data was collected by running ten thousand tests for each value of *n* from four to twenty (two thousand tests for CPII). The collected time data was graphed for a related growth trend for time complexity relation. As shown, the PII implementation follows a linear growth trend, which is consistent with the domain chosen. Accuracy tests were performed for cases of the PII that did not terminate under *n* iterations, yielding an estimated 97% accuracy. The CPII implementation, however, follows a logarithmic growth trend. The CPII performs less operations than the PII per iteration, but the speed of the PII can become similar to that of the CPII through the implementation of logarithmic operations. However, through testing, the CPII always terminated with a stable matching and no unstable pairings. Thus, the CPII, through implementation testing, does converge and holds a logarithmic-growth run time complexity when compared to the PII.

FUTURE WORKS

Regarding the CPII, proving an average growth trend for the algorithm through the testing of large vales of *n* is the main goal. Similarly, other algorithms have been developed to improve upon the PII, such as the Parallel Iterative Improvement: Smart Initiation and Cycle Detection (PII-SC), without implementation-based results. Also, attempts at applying parallel considerations to the stable roommates problem are planned.

ACKNOWLEDGMENTS

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