

# WBDev Tilted Beta and Tilted Beta-Binomial Distributions Documentation

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These new stochastic components for WinBUGS implement the tilted beta and tilted beta-binomial distributions. The components make use of the WBDev capabilities (Lunn 2003) of WinBUGS 1.4 and higher versions. Please follow the installation instructions at the WBDev website. Install the WBDev patch and the WBDev shared components, then place the supplied code in the correct locations and modify the `distributions.odc` file with the below additions.

## Overview

The tilted beta distribution (Hahn and López Martín 2015) has PDF

$$p(x|\alpha, \beta, v, \theta) = \begin{cases} (1 - \theta)(2 - 2v + (4v - 2)x) + \theta \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1} (1 - x)^{\beta-1} & \text{if } 0 \leq x \leq 1, \\ 0 & \text{otherwise.} \end{cases} \quad (1)$$

with  $\alpha > 0$ ,  $\beta > 0$ ,  $v \in [0, 1]$ , and  $\theta \in [0, 1]$ . Here  $\alpha$  and  $\beta$  are the standard parameters of the beta distribution,  $v$  governs the tilt, and  $\theta$  is a mixture weight for the mixture.

If instead  $x$  is the observed number of successes in  $n$  binomial trials, and if the binomial probability of success follows (1), then  $x$  has the tilted beta-binomial distribution with PMF

$$p(x|n, \alpha, \beta, v, \theta) = (1 - \theta) \frac{2(x(2v - 1) - nv + n + 1)}{(n + 2)(n + 1)} + \theta \binom{n}{x} \left( \frac{B(x + \alpha, n - x + \beta)}{B(\alpha, \beta)} \right), \quad (2)$$

where  $x$  and  $n$  are integers such that  $0 \leq x \leq n$  and where  $B(\cdot, \cdot)$  is the Beta function.

Examples of the new components' usage are

```
x[i] ~ dtilted.beta(alpha,beta,v,theta)
```

and

```
x[i] ~ dtilted.betabinomial(alpha,beta,v,theta,n[i])
```

where quantities are as defined above.

## Important Usage Note Regarding Censored Variables

At the time of writing, censoring is **NOT** supported in the above constructs. WinBUGS has the `I[lower, upper]` notation that can be used for censored variables, but this is NOT supported here. Hence do not use the `I[lower, upper]` notation with these constructs.

## References

- Eugene D. Hahn and María del Mar López Martín. Robust project management with the tilted beta distribution. *SORT-Statistics and Operations Research Transactions*, 39(2):253–272, 2015.
- David Lunn. WinBUGS development interface (WBDev). *ISBA Bulletin*, 10(3):10–11, 2003.