Investigating Linear Relationships from Multiple Perspectives

Intended Audience Middle School Teachers of Algebra and Data Analysis

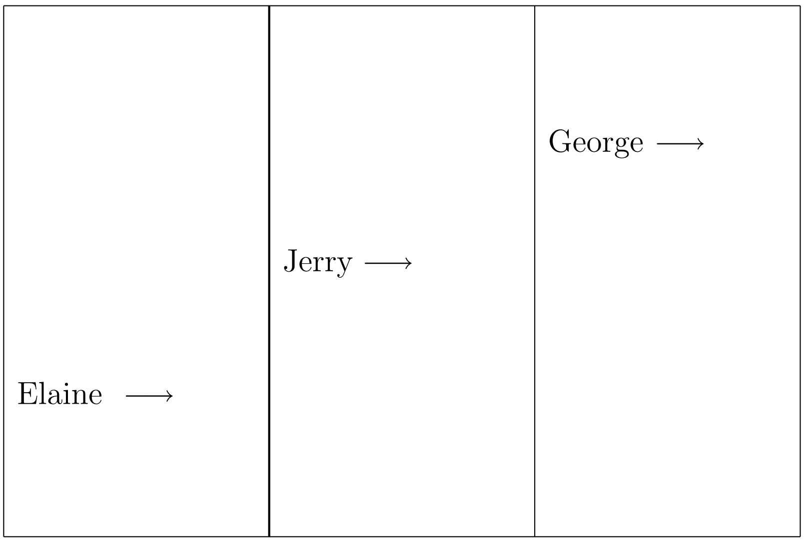
Goals: To model linear relationships from multiple perspectives, including numerically, graphically, and algebraically. To use these these different perspectives as a means to deepen understanding of linear relationships and make connections between the different approaches.

Prerequisites

1. Basic knowledge of the *Balanced Scale* virtual manipulative from the National Virtual Manipulative Library - Algebra Grades 6-8.
2. Ability to solve linear equations using your graphing calculator.
3. A very basic knowledge of Algebra Tiles.

The Scenario

Suppose that Jerry, Elaine, and George start a race as shown in the following diagram:



Starting Line

Jerry starts at the starting line. Since Elaine is a little faster, she starts a little behind Jerry. Since George is the slowest of the group, he starts a little ahead of Jerry. Suppose the starting position and speed of each of the runners are as follows:

Runner	**Starting Position**	**Speed of Runner**
Jerry	$y = 0$ meters	4 meters per second
George	$y = 2$ meters	3 meters per second
Elaine	$y = -2$ meters	5 meters per second

Activity I. Multiple Approaches to the Same Problem

1. **Table Approach:** Set up a numerical table based on the chart above that gives the position of each runner after the first 10 seconds. Use 1 second increments in your table.

2. **Symbolic/Algebraic Representation:** Use the data from your table in # 1 to construct equations that represent the position of each runner (in meters from the starting line) in terms of the number of seconds run.

3. **Physical Models:**

a. Use the **Balanced Scale** virtual manipulative (from the National Virtual Manipulative Library - Algebra Grades 6-8 and your equations from Question 2 to figure out when each pair of racers are in the same position. This virtual manipulative can be obtained at the following address:

http://nlvm.usu.edu/en/nav/vlibrary.html

b. **Algebra Tiles**: Now we are going to use algebra tiles to solve the same equations. Let us first solve the equation $4x = 3x + 2$, which is one of the equations you should have solved in part (a). To do this, perform the following steps:

i) Start by finding seven of the x-tiles and two of the unit tiles.

ii) Create two separate work areas - one "$4x$" area and one "$3x + 2$" area. (The areas are for the two sides of the equation are separate to avoid confusion about which side of the equation a given tile belongs to).

iii) In the first work area place four of the x-tiles. In the second area place three of the x-tiles and two of the unit tiles. Caution: Make sure not to think of the physical length of the x-tile as representing the solution to this equation. It does not.

iv) Next you will need have access to several additional unit tiles. For each x-tile, place a single unit tile over the x-tile. Make sure you do this in both work areas (i.e. on both sides of the equation).

v) Now we can see if $x = 1$ is a solution to this equation. There are 4 unit tiles in the "$4x$" work area and 3+2=5 unit tiles in the "$3x + 2$" work area. Thus, $x = 1$ is not a solution to our equation.

vi). Place another unit tile over each x-tile in both work areas. Count the total number of unit tiles in both areas. You should obtain 4x2=8 in one area and 3x2+2=8 in the other. So $x = 2$ is the solution to this equation.

Remark: In doing this activity we are assuming that our equation has an integer solution. This is not always the case when solving linear equations.

Exercise: Repeat this process to solve the other two equations using algebra tiles. Note that the equation $5x - 2$ requires "negative" unit tiles. One way to handle this is to use the red side of a unit tile to represent -1 and the blue side to represent +1. So when you create your work areas, two red -unit tiles and two blue unit tiles will cancel each other out.

4. **Integrating Technology:** Use your graphing calculator to repeat # 3. Discuss the significance of the slopes of your graphs as well as the y-intercepts. See your handout "An Introduction to Graphing Calculator Basics" for help with graphing linear equations, solving linear equations, zooming, etc.

5. **Writing:** Write a paragraph to describe who is in first, second, and third place for various time intervals and why. Once one runner passes another, can they ever be in the same position again? Why or why not?

Activity II. Repeat *I* for the following data:

Runner	**Starting Position**	**Speed of Runner**
Jerry	$y = 0$ meters	4 meters per second
George	$y = 4$ meters	3 meters per second
Elaine	$y = -6$ meters	5 meters per second

Activity III. Extensions: Non-integer Solutions

Repeat *I* for the data shown below. Note that not all solutions to the equations you solve will be integers. In the non-integer case, the *Balanced Scales* virtual manipulative cannot be used. However, your graphing calculator will still be effective for modeling the equations and corresponding solutions.

Runner	**Starting Position**	**Speed of Runner**
Jerry	$y = 0$ meters	4 meters per second
George	$y = 4$ meters	3 meters per second
Elaine	$y = -7$ meters	5 meters per second

Question: How could you tell using algebra tiles that a linear equation does not have an integer solution?